# *the family* MATH companion

ARITHMETIC—THE FOUNDATION OF MATH

*Ruth C. Sun*

# The Family MATH Companion

Arithmetic - The Foundation of Math

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Ruth C. Sun

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# Preface

More than six years ago, in response to a need, I started math clubs for students from second grade to eighth grade, teaching arithmetic, pre-algebra, and algebra. It led me to ask and analyze why many students did poorly in math and so few were ready for algebra at eighth grade. The following are some of the findings, I believe, that contribute to the poor performance in math.

* Math study seems to jump from one topic to another with little logical sequence. Students do not know what is essential and what is not.
* Students learn how (the skills), but not what (the concept) and why (the reason) Most of the students do not know how numbers are related.
* Many students learn little at the end of the school year, and lose the "little" they have learned during the long summer vacation.
* When the fall comes, students are back to "square one" and they are tired of going over the same old stuff again.

Failure in math seems to affect a child's self-image more than any other subject. It undermines a child's confidence. It is to correct these problems that "The Family Math Companion," a reference book, was written. The book deals exclusively with three positive numbers and four operations to show "what arithmetic is" and "how it works" in a systematic, logical way. It is intended to lay a solid foundation upon which the students will be able to move to a higher level of math. For that reason, graphing, geometry, etc., are excluded.

The book is intended to serve:

* ***Parents.*** The book is complementary to math books used in school. It is a tool which will give the parents the needed confidence to assist their children in math work at home. We know "failure tends to breed failure; success breeds success." Failure in math is something that can be prevented if the home works together with the school. Mathematics is a unique subject, once a student gets behind, it is very difficult for him or her to catch up.

* ***High school students.*** The book can help high school students who are lost in the maze of math to find a way out. Particularly, it gives an exhaustive treatment of "factors, multiples, and the related concepts and skills" (pp.277-328) which are absolutely essential to do fractions.

* ***College-bound students*** The book provides an excellent review for those who are preparing to pursue a higher level of education.

It is fashionable today to talk about algebra and to make algebra relevant to students. But the question is "How can a student study algebra without the solid arithmetic foundation?" If students are expected to study algebra, then make sure they learn first how to add, subtract, multiply, and divide. First things come first. **Back to the basics!**

I would like to thank the school administrators and teachers of school districts in the Chicagoland area for their assistance in my research. Dr Harry Agabedis and Ms. Anna Horn Kreske deserve special thanks for their support and encouragement and special thanks also to Mrs. June Osborne. Thanks to SSC in South Holland, Illinois for the use of their laser printer and technical assistance from some of their staff Thanks also to Mr. Jerry Spehar for his technical assistance. Finally, thanks to Mr George Stone for some practical suggestions.

Ruth C Sun
April, 1996

## How To Use This Book

**The Family Math Companion** does not contain exercises because it is a **know-how** book. Since it is complementary to the school math books, you can find plenty of exercises in those types of textbooks.

Make Good Use of the Unique Features of This Book:

* Each page illustrates a topic with a step-by-step explanation. Read each word carefully to be sure you understand
* Read those pages as indicated in the cross references. It is the "prior knowledge" you need to understand the topic you are studying.
* The concepts or skills that are easy to get confused are placed side by side for comparison. Study them until you understand the differences.
* Pay attention to the words printed in bold letters. They are key words, or concepts, or rules that you should memorize, once you understand.
* To obtain the most benefit, take time to read the complete "Part" or at least the "Introduction" and the entire "Section" related to the problem you are working By so doing, you will learn mathematics in a systematic way

General Procedure for Using the Book:

1st. Determine the nature of the problem you are working by asking yourself:
   a) Is the problem related to "Numbers & Concepts" or "Operations"?
   b) If operation, to which number is it related - whole numbers? decimals? or Fraction?
   c) Then, which operation - addition? subtraction? multiplication? or division?

2nd. Turn to the "Outline" on page 1 Find the "Part" (Part I, II, etc.) and then the "Section" (A, B, etc.) that the problem is under Take note of the pages given.

3rd Turn to the "Section" as indicated by the pages. Find the "Table of Contents", then going down the list, you will find the topic that is related to the problem you are working on.

Example 1 Subtract 7003 – 129

1st. Determine the nature of the problem. It is a whole number operation subtracting across zero.

2nd Turn to the "Outline" on page 1 Find "Whole Number Operations" (Part II), under that you will find the section "C Subtraction" with the page numbers of 147 - 164.

3rd. Then turn to "Table of Contents" for the subtraction of whole numbers on pages 149 Going down the list you will find the topic "Subtracting Across Zeros" with the page number of 163 Turn to page 163 for the information you need

Example 2. Divide 36 ÷ 1.2.

1st. Determine the nature of the problem. It is a division operation - dividing a whole number by a decimal.

2nd. Turn to the "Outline" on page 1 Find "Decimal Operations" (Part III), under that you will find the section "E. Division" with the page numbers of 245-262.

3rd. Then turn to "Table of Contents" for the division of decimals on page 247 Going down the list you will find the topic "Dividing Whole Numbers By Decimals" with the page number 256. Turn to page 256 for the information you need.

To Do Well in Math Requires:

Learning the mathematical terms and symbols.

Having the prior knowledge of related concepts and skills.

-- A lot of practice.

Of course, you need discipline and concentration Read each word carefully, because mathematical statements are made up of compact and precise sentences.

# OUTLINE



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***"The Family Math Companion"*** **- an educational tool every family should have. It is written to enable the parents to become math tutors at home. The book will lay a solid arithmetic foundation upon which a student will be able to move to a higher level of math.**
**This is a reference book with the following features:**

- ***It is user friendly and easy to understand.***
- ***It is illustrative. Every page explains mainly one concept or one skill.***
- ***It is systematic and logical. It explains why as well as what and how.***
- ***It gives extensive cross references to show interconnection of math.***
- ***It states the important points repeatedly to call attention of the readers.***

*Ruth C. Sun*
**For many years, had math clubs for students from second grade to eighth grade, teaching arithmetic, pre-algebra, and algebra. She received her M.A. from Wheaton Graduate School and is also the author of *"Personal Bible Study. A How To"* She is married to a scientist.**

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