the family MATH companion

ARITHMETIC—THE FOUNDATION OF MATH

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Part V. Ratios, Proportions, Percents

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Ratios, Equal Ratios (See also "Equivalent Fractions" p.300)

A ratio is a **comparison of two numbers or quantities.** It implies a comparison by division. The following is the ratio of 8 boys to 2 girls.

a) 8 **to** 2

b) 8 : 2

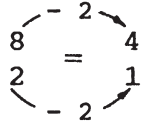
c) $\frac{8}{2}$ ← 8 = 1st term, 2 = 2nd term

* A ratio has **two terms: "1st term to 2nd term"**
* A ratio can be expressed in **three different ways** as shown on the left: **in word, in colon,** and **in fraction.** (Fraction form is commonly used.)
* All three forms are read **"8 to 2."**

Order is important in writing a ratio. **The quantity named first in statement is always the first term.** For example, there are 8 boys and 2 girls:

What is the ratio of **boys** to girls?	**8** to 2	or	**8**/2	(boys first)
What is the ratio of **girls** to boys?	**2** to 8	or	**2**/8	(girls first)

Equal Ratios - Ratios like fractions are usually written in lowest terms.



8 : 2 = 4 : 1

We simplify ratios, write equal ratios,and change ratios to a common denominator like we do with fractions. But we can not change a ratio such as 4/1 or 5/2 to a whole number or a mixed number.

Ratios & Rates, Unit Rate & Unit Price

A rate is a ratio **comparing two quantities in different kinds of units.** You know the comparison is a rate when it uses words like **"per" "for" "each."** The following compares the differences between ratios and rates.

Ratio $\frac{3 \text{ points}}{4 \text{ points}}$ — score of Sue / — score of Tom ["3 **to** 4" / "3 : 4"]

In a ratio, the terms are of the same kind or in the same unit.

Rate $\frac{60 \text{ miles}}{1 \text{ hour}}$ — distance / — time [60 **miles per (1)hr** / 60 **miles : 1 hr**]

In a rate, the terms are always in different units.

In ratios, same units cancel out. But in rates, different units are written as part of the rate.

Unit Rate & Unit Price - In unit rate or unit price **the second term is 1 unit.** Unit Rate is the rate that **compares quantity to 1 unit.** Unit price is the rate that compares price to 1 unit.

Rate: "120 pages in 3 hours"
Unit Rate: "40 pages/1 hr"

Price: "$9 for 2 lbs"
Unit Price: "$4.50/1 lb"

To find the unit rate or unit price, divide the number in the first term by the number in the second term:
120 ÷ 3 = 40; 9 ÷ 2 = 4.5

Ratios & Proportions

A ratio is a comparison of two numbers. A proportion is **an equation showing that two ratios are equal** and it can be written in the following ways:

a) 48 : 12 = 4 : 1

b) $\frac{48}{12} = \frac{4}{1}$ ← **The 1 can not be dropped in a ratio.**

Both read as "48 is to 12 as 4 is to 1"

A ratio has two terms but a proportion has 4 terms. We call
- the **1st** and **4th terms** (last term) the **extremes** of the proportion,
- the **2nd** and **3rd terms** (two middle terms) the **means** of the proportion.

In a true proportion: **The product of the means equals the product of the extremes.**

(2nd term) x (3rd term) = (1st term) x (4th term)

a) 48 : 12 = 4 : 1 (means: 12 and 4; extremes: 48 and 1)

b) $\frac{\text{1st term (extreme)}}{\text{2nd term (means)}} = \frac{\text{3rd term (mean)}}{\text{4th term (extreme)}}$ → $\frac{48}{12} = \frac{4}{1}$

We can determine whether two ratios are equal by following methods:
1. **Reduce each ratio to lowest terms and compare.** (See p309)
2. **Use the cross products.** (See next page)

Cross Products - A Test of Equal Ratios or Equivalent Fractions (See also p.303)

We can use cross products (also called cross multiplication) to determine whether two ratios or two fractions are equal. We know in a true proportion, **the product of the extremes equals the product of the means.**

① ②

$\frac{20}{50} \overset{?}{\times} \frac{18}{45}$

① $20 \times 45 = 900$ ← **The products of the extremes**

② $50 \times 18 = 900$ ← **The products of the means**

$\frac{20}{50} = \frac{18}{45}$ Since the cross products are equal, the two ratios are equal. Thus, 25/50 = 18/45 is a proportion.

We can use proportion and cross products to **find the value of the "unknown"** term if we know the other three terms. Follow the steps given below:

① $\frac{14}{21} = \frac{6}{n}$ ② $14 \times n = 21 \times 6$ ③ $n = \frac{21 \times 6}{14} = 9$ ④ $\frac{14}{21} = \frac{2}{3}$, $\frac{6}{9} = \frac{2}{3}$

① **Set up a proportion:** with an unknown term "n" in one of the ratio.
② **Cross multiply:** the products of the extremes = the products of the means.
③ **Solve for n:** divide the product of the means by the given extreme.
④ **Check:** reduce each ratio to lowest terms and compare. Check (√).

Setting Up Proportions In Different Ways (Review first p.374)

In the following, we use the proportion **4 : 2 = 14 : 7** as an example to show that there are **more than one way** of setting up a proportion. We can use the following formula to test whether each proportion is a true proportion:

2nd term x 3rd term = 1st term x 4th term

(a) 1st term $\frac{4}{2} \overset{?}{=} \frac{14}{7}$ 2nd term

3rd term ⟶ 2 x 14 = 28

4th term ⟶ 4 x 7 = 28

true proportion

(b) 2nd term $\frac{2}{4} \overset{?}{=} \frac{7}{14}$ 1st term

4th term ⟶ 4 x 7 = 28

3rd term ⟶ 2 x 14 = 28

true proportion

(c) 2nd term $\frac{2}{7} \overset{?}{=} \frac{4}{14}$ 4th term

1st term ⟶ 7 x 4 = 28

3rd term ⟶ 2 x 14 = 28

true proportion

(d) 4th term $\frac{7}{2} \overset{?}{=} \frac{14}{4}$ 2nd term

3rd term ⟶ 2 x 14 = 28

1st term ⟶ 7 x 4 = 28

true proportion

Scale Drawings (Review "Proportions" "Cross Products" p.374)

A scale drawing reduces the size of an actual object proportionately so that the object appears in the same shape but in a much smaller size. Scale drawings are used to make maps, draw blue prints for buildings, cars, plane, etc. **It utilizes the concepts of proportions and cross multiplication.**

Here is how to use proportion to find the actual distance of the city 13 inches away on the map, if the scale is 1 inch for 20 miles.

$$\frac{\text{distance in drawing (inch)}}{\text{actual distance (mile)}} \quad \frac{1}{20} \quad = \quad \frac{13}{n} \quad \frac{\text{distance in drawing (inch)}}{\text{actual distance (mile)}}$$

The units of the second ratio must correspond to the units of the first ratio. (See p.373)

cross multiply $1 \times n = 13 \times 20$ $n = 260$ miles

check $1 \times 260 = 260$; $13 \times 20 = 260$ or $13/260 = 1/20$

Step 1. Set up a proportion. Use **the given scale** to write the **first ratio:** 1/20 and use n to represent the unknown distance.
Step 2. Find cross products. Solve for n.
Step 3. Check. Substitute 260 for n and multiply or simplify. Check (√)
The answer: The city is 260 miles away.

Meanings of Percents and The Percent Symbol (Sign) (%)

The word percent comes from **"per"** and **"centum." "Centum" means 100.** So, percent means "by the hundredths" (the number of hundredths of a number), "per 100," "out of 100," "on the basis of 100," "compared with 100," or "divided by 100." The symbol for percent is %. Study the following carefully and find what the percent symbol % stands for:

$$5\% = \frac{5}{100} = 5 \div 100 = 5 \times \frac{1}{100},$$

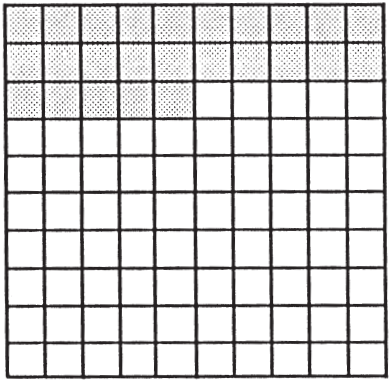
$$1\ \% = \frac{1}{100} = 1 \div 100 = 0.01$$

$$100\% = \frac{100}{100} = 100 \div 100 = 1$$

Points To Remember: **(See "Shortcut..." p.390)**

* **To drop the % sign means to divide the number by 100** or to multiply by its reciprocal 1/100 (= 0.01). **(See "Reciprocal" p.326)**
* **To add the % sign means to divide the number by 100.** Therefore, you first multiply the number by 100 before you add the % (= to multiply by 100/100).
* **Any number with the % sign should be changed** to a decimal, or a fraction, or a ratio before it is used in computation.

Relating Percents to Ratios, Fractions, & Decimals



On the left is a 10-by-10 grid, 25 out of 100 squares are shaded. When we compare the shaded squares to the total of 100 squares, we can express the relation as:

* **a percent:** 25% (25 per 100)
* **a ratio:** 25:100 or 25/100
* **a fraction:** 25/100
* **a decimal:** 0.25

The example shows the basic relationships that exist among percent, ratio, fraction, and decimal. They are different forms of writing the same number. **(See p.98)** We can say:

- Percent is a special kind of **ratio** which always **compares with a number 100.**
  (e.g.) 25% means "25 compared with 100" -- 25:100 or 25/100
- Percent is a special kind of **fraction** whose **denominator is always 100.**
  (e.g.) 25% means "25 out of 100" -- 25/100
- Percent is a special kind of **decimal** with the place value of **"hundredths."**
  (e.g.) 25% means "25 hundredths (of 100)" -- 0.25

Percents and Decimal/Fraction Equivalents (Review also p.98)

Since percents, decimals, and fractions are different forms of writing the same number, you should not only know how to change from one form to another but also remember the equivalent forms of the most commonly used percents like the one listed below.

Percent	Decimal Equivalent	Fraction Equivalent
1%	0.01	1/100
5%	0.05	1/20
10%	0.1	1/10
20%	0.2	1/5
25%	0.25	1/4

Percent	Decimal Equivalent	Fraction Equivalent
40%	0.4	2/5
50%	0.5	1/2
60%	0.6	3/5
80%	0.8	4/5
100%	1	1

Note: To change **a mixed number percent,** $12\frac{1}{2}\%$, to a decimal, first write it as a **sum** of a whole number percent and a fraction percent:

$12\frac{1}{2}\% = 12\% + 1/2\% = 12\% + 0.5\% = 0.12 + 0.005 = 0.125$

Fractions and Decimal/Percent Equivalents

Also for your quick reference, here are the decimal/percent equivalents of some of the common fractions which you are likely to encounter in working with numbers.

Common Fraction	Decimal Equivalent	Percent Equivalent
1/2	0.5	50%
1/3	0.33	$33\frac{1}{3}$%
1/4	0.25	25%
1/5	0.2	20%
1/6	0.16	$16\frac{2}{3}$%
1/8	0.125	$12\frac{1}{2}$%

Common Fraction	Decimal Equivalent	Percent Equivalent
2/3	0.66	$66\frac{2}{3}$%
3/4	0.75	75%
5/6	0.83	$83\frac{1}{3}$%
3/8	0.375	$37\frac{1}{2}$%
5/8	0.625	$62\frac{1}{2}$%
7/8	0.375	$37\frac{1}{2}$%

Example: Find $16\frac{2}{3}$% of 240.

1/6 x 240 = 40 $16\frac{2}{3}$% = 1/6 and the word "of" means "times."

Percents, Decimals, & Fractions

Keep in mind that decimals and percents are **two special types** of fractions which differ from the common fractions The differences are found in their denominators:

$\frac{2}{5}$ $\frac{3}{4}$ $\frac{1}{25}$ ←

In **Common Fractions,** the denominators are any non-zero number except the powers of 10.

$0.04 = \frac{4}{10}$ $0.25 = \frac{25}{100}$ ←

In **Decimals**, the denominators are the powers of 10. Instead of writ- the denominators, they are indi- cated by the position of the digit. **(See p.98)**

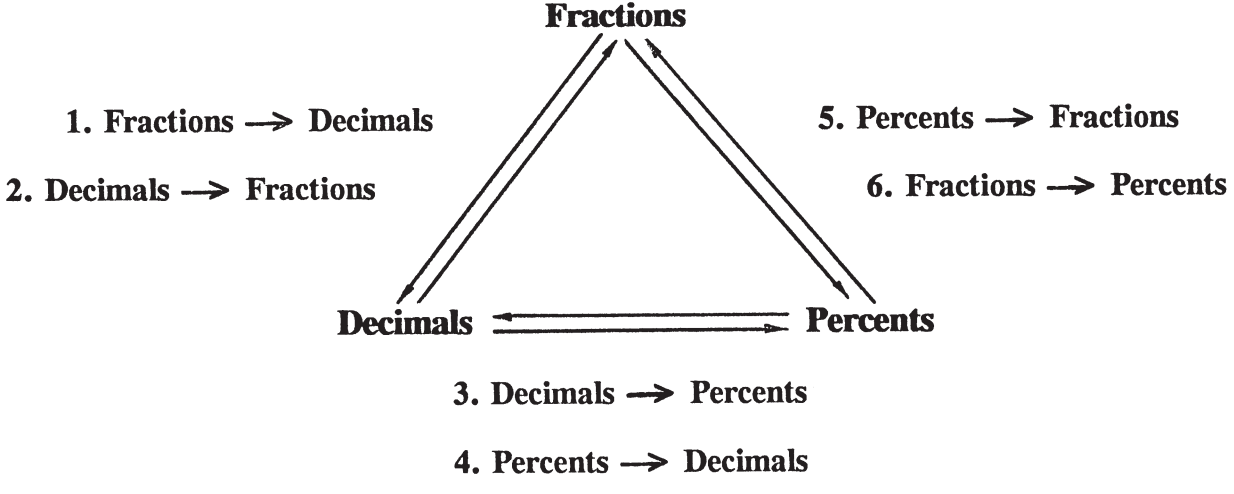
$5\% = \frac{5}{100}$ $20\% = \frac{20}{100}$ ←

In **Percents,** the denominators are always 100. Instead of writing the denominator, they use a percent sign % (% = 1/100). **(See p.378)**

Fractions are always written in the lowest terms. But when a fraction represents a percent, you can not simplify it. For example, you can not write 5/100 & 20/100 above as 1/20 & 1/5.

Interchanging Among Fractions, Decimals, & Percents

Since fractions, decimals, and percents are **different forms of writing the same number,** it is important that you know how to change from one form to another. The following shows the **six possible interchanges** and "How-To" examples are given on the successive pages.



1. Changing Fractions to Decimals (Review first p.98)

How to change a fraction to a decimal depends on the denominator of the fraction as shown below. If you are not sure, **you can always change a fraction to a decimal by dividing the numerator by the denominator.**

$\frac{2}{10} = 0.2$ $\frac{25}{100} = 0.25$

*** For Decimal Fractions**

Write directly as a decimal the way the fraction is read (See p.98)

Clue: The decimal places should equal the number of zeros in the denominator

$\frac{3}{4} = 3 \div 4 = 0.75$

*** For Common Fractions**

Division changes any fraction to a decimal. But the result can be:

a) $\frac{3}{4} \overset{\times 25}{=} \frac{75}{100} = 0.75$ (x 25 above and below)

a) A terminating decimal - If the given fraction can be changed to a **equivalent decimal fraction.** Then write as a decimal. (p.266)

b) $\frac{1}{3} = 1 \div 3 = 0.33...$

b) A repeating decimal - If the given denominator **does not** divide evenly into 10, 100, etc. (p.267)

2. Changing Decimals to Fractions

Remember: The denominator of a decimal is always a power of 10 (p.382)

$$0.04 = \frac{4}{100} = \frac{1}{25}$$

hundredths — reduce

$$2.105 = 2 + 0.105$$

$$2 + 0.105 = 2 + \frac{105}{1000}$$

$$2 + \frac{105}{1000} = 2\,\frac{105}{1000} = 2\,\frac{21}{200}$$

*** For Decimal Less Than 1 (< 1):**

1st. Write the **decimal as the numerator.** (Drop zero before whole number p.66)

2nd. Write **the place value of the last digit as the denominator.**

3rd. Reduce to lowest terms.

*** For Decimal Greater Than 1 (> 1):**

1st. Write the mixed decimal **as the sum of whole number and decimal** (p.224)

2nd. Keep the whole number part. But **change the decimal part to fraction** following the method given above.

3rd. Write as a mixed numbers with "+" **omitted.** Simplify the fraction.

Suggestion: Read **"Decimals & Decimal Fractions"** (p 98) & **"Changing Fractions to Decimals"** (p.266) together with last page and this page.

3. Changing Decimals to Percents

Remember: Percent fractions always have 100 as the denominator.

Change 0.5 to a percent.

$$0.5 \times \frac{100}{100} = \frac{50}{100}$$

$$\frac{50}{100} = 50 \times \frac{1}{100} = 50\%$$

$$0.5 = \frac{5}{10} \overset{\times 10}{\underset{\times 10}{}} \frac{50}{100} = 50\%$$

$$0.5 \longrightarrow 0.50. \longrightarrow 50\%$$

Method 1. Use "1" Property

1st. Multiply 0.5 by 100/100 (= 1) changing the decimal to a fraction with the denominator of 100.

2nd. Write the fraction as the numerator times 1/100: 50 x 1/100
(A reverse process of multiplying fraction.)

3rd. Replace 1/100 with %. (See p.378)

Method 2. Use Equivalent Fraction

1st. Write 0.5 as a decimal fraction.

2nd. Multiply 5/10 by 10/10 (= 1) changing it to a fraction with the denominator of 100.

3rd. Write as a percent. (See above)

Shortcut: Move the decimal point two places to the right and add the %. (See p.391)

4. Changing Percents to Decimals

Remember: $\% = (1)\% = 1/100 = 0.01$ (p.378)

$6\% = 6 \times \frac{1}{100} = \frac{6}{100}$

$\frac{6}{100} = 6 \div 100 = 0.06$

$6\% = 6 \times 0.01 = 0.06$

$212\% = \frac{212}{100} = 2\frac{12}{100}$

$2\frac{12}{100} = 2.12$

$212\% = 212 \times 0.01 = 2.12$

$6\% \longrightarrow .06 \longrightarrow 0.06$

*** For Percents Less than 10%**

Method 1. Change the % to 1/100

Drop the % and multiply by 1/100 changing the percent to a fraction with a denominator of 100. Then divide.

Method 2. Change the % to 0.01

Drop the % and multiply it by 0.01.

*** For Percents Greater than 100%**

Method 1. Change the % to 1/100

First, change the percent to a fraction. Then, write improper fraction as a mixed number, and then as a mixed decimal.

Method 2. Change the % to 0.01

Shortcut: Drop the % and move the decimal point two places to the left (p.378)

5. Changing Percents To Fractions

Remember: % = (1)% = 1/100 = 0.01

Change 45% to a fraction.

$$45\% = 45 \times \frac{1}{100} = \frac{45}{100}$$

$$\frac{45}{100} = \frac{45 \div 5}{100 \div 5} = \frac{9}{20}$$

decimal fraction — common fraction

Shorter Way: Change to Fraction

1st. **Percent to fraction:** 45% = 45/100
45% means "45 out of 100" or "45 times 1/100".

2nd. **Fraction in lowest terms**
Change the decimal fraction to a common fraction.

Remember: Numbers end in 5 and 0 are divisible by 5 which means they have 5 as a common factor

$$45\% = 45 \times 0.01 = .45$$

$$.45 = \frac{45}{100} = \frac{9}{20}$$

Longer Way: Change to Decimal first

1st. **Percent to decimal:** 45% = .45
Drop the % & move the decimal point two places to the left.
(See p.390)

2nd. **Decimal to decimal fraction:** 45/100

3rd. **Fraction in lowest terms.**

6. Changing Fractions to percents

$$\frac{3}{4} \xrightarrow{\times 25} \frac{75}{100} = 75\%$$

Method 1. Use Equivalent Fraction

Write an **equivalent fraction** with a denominator of 100.
Write as a percent. **(See #3, p.386)**

Use Method 1 if the given denominator is a factor of 100: 1, 2, 4, 5, 10, 20, 25, 50, 100.

$$\frac{5}{8} = 5 \div 8 = 0.625$$

$$0.625 \times \frac{100}{100} = 62.5\%$$

Method 2. Use Division

Change the fraction to a decimal by division. **(See #1, p.384)**
Multiply the decimal by **100/100 (= 1)**.
Write as a percent. **(See #3, p.386)**

By using Method 2, we can change any type of fraction to a percent.

$$\frac{5}{8} \overset{?}{=} \frac{n}{100} \longrightarrow 8 \times n \longrightarrow 5 \times 100$$

$$8 \times n = 5 \times 100$$
$$n = 500 \div 8 = 62.5\%$$

Method 3. Use Proportion **(See p.374)**

Write **an equation** making 5/8 equal to **n/100 (an unknown percent).**
Find the **cross products** and solve for n **(n equals percent).**

Shortcut For Changing Percents To Decimals (Review first p.378)

Shortcut: "*Drop the %* and move the decimal point two places *to the left.*"

Here is the reason why: [Remember: % = (1)% = 1/100 = 0.01]

$$[\% = 1/100] \quad 45\% = 45 \times \frac{1}{100} \;(1) = \frac{45}{100} \;(2) = 45 \div 100 \;(3) = 0.45 \;(4)$$

(1) To drop the % means to multiply the number by 1/100: 45 x 1/100
(2) To multiply the number by 1/100 change it to a fraction with a denominator of 100: 45/100
(3) **Fractions means division:** 45 ÷ 100
(4) **To divide by 100,** move the decimal point two places *to the left:*

45 ——— .4 5 ———→ .45 (or 0.45)

[% = 0.01] 45% = 45 **x 0.01** = .45 (or 0.45)

Or, drop the % means to **multiply** the number **by 0.01** as shown above.

As you see, ***"to drop the %"*** means either *to divide* **the number by 100** or *to multiply* **it by 0.01.** In either case, **move the decimal point *two places to the left.***

Shortcut For Changing Decimals To Percents (Review first p.228)

Shortcut: "Move the decimal point two places *to the right* and *add the %.*"

Here is the reason why: [Remember: Percents always has a denominator of 100.]

$$0.72 = 0.72 \times \underset{(1)}{\frac{100}{100}} = \underset{(2)}{\frac{72}{100}} = 72 \times \underset{(3)}{\frac{1}{100}} = \underset{(4)}{72\%}$$

(1) Multiply the number by 100/100 (= 1): 0.72 x 100/100
(2) Multiply the number by 100/100 change it to a percent fraction with a denominator of 100: 72/100
(3) Write the fraction as the numerator time 1/100: 72 x 1/100
(4) Write 1/100 as the % and add it to the number: 72%

0.72 ⟶ 0.72 x 100 = 72 ⟶ 72%

Or, multiply the number by 100 first and then add the % (= x 1/100)

Do you see that "to move the decimal point *two places to the right"* means *to multiply* the number by 100. And "to add the %" means to multiply it again by 1/100. It is the same as multiplying by 100/100 (= 1).

Changing Mixed Number Percents to Fraction/Decimal Equivalents

Changing 87 1/2% to A Fraction:

$$87\frac{1}{2}\% \overset{(1)}{=} 87\frac{1}{2} \div 100 = \frac{\cancel{175}^{7}}{2} \times \frac{1}{\cancel{100}_{4}} \overset{(2)}{=} \frac{7}{8} \overset{(3)}{}$$

87 1/2% can be written as a complex fraction: 87 1/2 over 100.

(1) Drop the % and *divide* the mixed numbers by 100: 87 1/2 ÷ 100
(2) ***To divide by 100*** is the same as ***to multiply by its reciprocal 1/100.***
Change the mixed number to an improper fraction and **cancel first.**
(3) 87 1/2 % equals 7/8. **(Check: 7/8 = 7 ÷ 8 = 0.875 = 87 1/2 %)**

Changing 87 1/2% to A Decimal:

$$87\ 1/2\% \overset{(1)}{=} 87\% + 1/2\% = 87\% + 0.5\% \overset{(2)}{=} .87 + .005 \overset{(3)}{=} 0.875 \overset{(4)}{}$$

(1) Write the mixed number percent as the **sum** of the whole number percent and the fractional percent.
(2) Change the fractional percent to decimal percent.(1/2 = 1 ÷ 2 = 0.5)
(3) Write both percents as decimals.(See "Shortcut" p.390)
(4) 87 1/2% equals 0.875. **(Check: 0.875 = 875/1000 = 7/8)**

A General Percent Equation

If we use **R** (ratio or percentage), **T** (total or base), and **P** (part) to represent three terms or quantities in a percent problem, a general statement of percents and its equation will be as follows:

General statement: "Certain ratio **of** total **is** part."

Equation: **R** x **T** = **P**

The word "of" means "times" (x).
The word "is" means "equals" =).

The general statement means "Percentage times total equals its actual quantity "

A Proportion for Percent Problems (Review "Proportions" p.374)

One way of solving percent problems is using **equations.** Another is using a **proportion.** A proportion has four terms. In a percent problem one of the terms is always 100 and if two other terms are known, then a proportion can be used to find the fourth term. The following is the proportion used for solving percent problems. **Make sure the terms correspond!**

$$\frac{\text{part}}{\text{total}} = \frac{\mathbf{R}}{\mathbf{100}} = \frac{\mathbf{P}}{\mathbf{T}} = \frac{\text{part}}{\text{total}}$$

The proportion is read: "R compares with 100 is the same as P compares with T "

Three Basic Types of Percent Problems (Review first the previous page.)

There are three basic types of percent problems because a percent problem is made up of only three quantities. All three types of problems has to do with finding one of those three quantities when the other two are known. In the following ***n*** represents the unknown quantity.

Three Basic Types of Problem	Sample Questions
1. **Finding the part (P):** **R x T = *n*** 60% x 50 = n	* 60% of 50 is what number? * What (number) is 60% of 50? * Find 60% of 50.
2. **Finding the ratio (R):** ***n* x T = P** n x 50 = 30	* What percent of 50 is 30? * What percent is 30 of 50? * 30 is what percent of 50?
3. **Finding the total (T):** **R x *n* = P** 60% x n = 30	* 60% of what number is 30? * 30 is 60% of what number? * 30 is 60% of a number, find the number.

6. In general, the simplest way to find the part (P) (Type 1 problem) is to change the percent to a decimal or a fraction and multiply. Change to a fraction if cancellation can be used. **(See p.304)**

7. **A percent number (the ratio) should be changed to** a fraction, a decimal, or a ratio before it is used in computation. **(See p.378)**

8. In setting up a **proportion, the terms must correspond.** Each ratio compares the part with the total (or base). In a percent problem, one of the terms (base) is always 100. **(See p.393)**

9. To change a percent to decimal, drop the % and move the decimal point two places to the left or multiply the number by 0.01. **(p.390)**
   When changing a percent to a fraction, reduce it to lowest terms if necessary. **(See p.388)**
   For example, 50% = 0.50 = 0.5 or 50% = 50/100 = 1/2

10. To change a number (whole number or decimal) to a percent, multiply the number by 100 and add a percent sign (%).
    For example: 7 = 7 x 100 = 700%, & 0.24 = 0.24 x 100 = 24%

11. Utilize the **concepts of cancellation** **(See p.304)** and **reciprocal** **(See p.326)** whenever possible, it means less calculation.

Solving Type 1 Problems: Finding the Part (Read first p.394)

Problem: During the clearance sale, all the shoes at Z store are 25% off the regular prices. If the regular price of the pair of shoes which Dick bought is $84, how much did he save?

Analysis: The problem -- "25% (sale) of $84 (base price) is how much?"
The equation -- 25/100 x 84 = n and n = 25/100 x 84
The question asked -- "How much did Dick save?"

Clue: If you remember that **25% = 1/4** (25 is a quarter of 100) and if you understand the concept of cancellation (see p.304), the problem can be solved with little calculation. See Method 1.

Method 1. Using Equation - Changing the percent to a fraction

$n = \frac{1}{\cancel{4}_{1} } \times \overset{21}{\cancel{\$84}}$	Change 25% to 1/4 and multiply. Use cancellation.
n = $21	Ans. Dick saved $21.

To know how much Dick paid for the pair of shoes, subtract $21 from $84 (84 - 21 = 63). Dick paid $63 for the shoes.

Method 2. Using Equation - Changing the percent to a decimal

$n = 0.25 \times \$84$	Change 25% to 0.25 and calculate.
$= \$21$	Ans. Dick saved \$21.

Method 3. Using Proportion - Changing the percent to a ratio

$\frac{1}{4} = \frac{n}{84}$	**1st.** Set up a proportion. Change 25% to 25/100 = 1/4 (lowest term).
$4 \times n = 1 \times 84$	**2nd.** Find the cross products. Have n on the left of the equation sign.
$\frac{4 \times n}{4} = \frac{\overset{21}{\cancel{84}}}{\cancel{4}_{1}}$	**3rd.** Divide both sides of the equation by 4 so n will be alone on the left side of equation sign.
$n = \$21$	Ans. Dick saved \$21.

To solve Type 1 problem, it is much simpler to use an equation method than a proportion method. It requires less computation.

Type 1 Problems include finding the sales tax and the service tip etc.

Solving Type 2 Problems: Finding the Ratio (or Percent) (Read first p.394)

Problem: In a math test, Judy got 32 problems correct out of 40. What percent of correct answers did she get?

Analysis: The problem -- "What percent of 40 (total) is 32 (part)?"
The equation -- n x 40 = 32 and n = 32/40 (32 - 40)
The question asked -- What % of correct answer did Judy get?

Clue: If you know the shortcut (See p.391), all you have to do is to divide the numbers and write the answer as a percent by moving the decimal point two places to the right and adding the %. See Method 1.

Method 1. Using Equation - Finding n and then write as n%

$$n\% = \frac{32}{40} = 0.8$$

1st. Divide. In algebra, division is written in a fraction form.

$$n\% = 0.8 \times \frac{100}{100}$$

$$= \frac{80}{100} = 80\%$$

2nd. The question is asking for a ratio. Multiply 0.8 with 100/100 (=1) changing the decimal to a fraction with a denominator of 100. Then write as a percent.
Ans. Judy got 80% correct.

Method 2. Using Equation - Changing the percent to a decimal

$(0.01)n = \frac{32}{40} = 0.8$ — **1st. Change the % to a decimal 0.01 (1/100), since n here stands for n%.**

$\frac{(0.01)n}{0.01} = \frac{0.8}{0\ 01}$ — **2nd.** Divide both sides by 0.01 and solve for n.

$n = 80\%$ — Ans. Judy got 80% correct.

Method 3. Using Proportion - Changing the percent to a ratio

$\frac{n}{100} = \frac{32}{40}$ — **1st.** Set up a proportion. Make sure each ratio is comparing the part to the total.

$n \times 40 = 100 \times 32$ — **2nd.** Find the cross products.

$\frac{n \times 40}{40} = \frac{\overset{5}{\cancel{100}} \times \overset{16}{\cancel{32}}}{\cancel{40}\ \cancel{2}\ \ 1}$ — **3rd.** Divide both sides by 40 and solve for n. Use cancellation.

$n = 80\%$ — Ans. Judy got 80% correct.

Solving Type 3 Problems: Finding the Total (or Base) (Read first p.394)

Problem: According to AA High School, 238 graduates of the class of 1993 went on to college. It represents 68% of the total graduates of the year. How many did graduate from AA High School that year?

Analysis: The problem -- "68% of what number is 238?"
The equation -- $.68 \times n = 238$ and $n = 238/.68$ $(238 \div 68\%)$
(In algebra, division is written in fraction form.)
The question asked -- "How many graduated in 1993 from AA Hi?"

Clue: If you rewrite the percent as a decimal, you solve the problem in one step by dividing the part by the ratio. See Method 2.

Method 1. Using Equation - Changing the percent to a fraction.

$n = 238 \div \frac{68}{100}$	**1st.** Change the percent to a fraction.
$n = 238 \div 0.68$	**2nd.** Compute
$= 350$	Ans. 350 graduated.

Method 2. Using Equation - Changing the percent to a decimal

$n = 238 \div 0.68$ — **1st.** Change the percent to a decimal and divide 238 by the decimal.

$= 350$ — Ans. 350 graduated.

Method 3. Using Proportion - Changing the percent to a ratio

$\frac{17}{25} = \frac{238}{n}$ — **1st.** Set up a proportion. Change the percent to a ratio in lowest terms.

$17 \times n = 25 \times 238$ — **2nd.** Find the cross products, n on the left of the equation.

$\frac{17 \times n}{17} = \frac{25 \times 238}{17}$ — **3rd.** Divide both sides of the equation by 17 and solve for n.

$n = 350$ — Ans. The total graduates of the class 1993 was 350.

Percents & Money (Review also "Decimals & Money" p.59)

Since a dollar equals 100 cents, we can express it in the following ways:

% of a dollar	In Decimal	In Word	(In Fraction)
1% of a dollar	**$.01**	1 cent	(1/100 of $1.00)
8% of a dollar	**$.08**	8 cents	(8/100 of $1.00)
10% of a dollar	**$.10**	10 cents	(1/10 of $1.00)
25% of a dollar	**$.25**	25 cents	(1/4 of $1.00)
50% of a dollar	**$.50**	50 cents	(1/2 of $1.00)
75% of a dollar	**$.75**	75 cents	(3/4 of $1.00)
100% of a dollar	**$1.00**	100 cents	($1.00)
150% of a dollar	**$1.50**	150 cents	(1 1/2 of $1.00)

Note: "8% sale tax" means you pay an extra 8 cents for every dollar you spend.

Summary
(Ratios, Proportions, & Percents)

* A ratio is a comparison of two numbers or quantities of the same kind of units. A rate is a comparison of two qualities in different kinds of units.
* A proportion is an equation which shows that two ratios are equal. Cross products is a method used to test whether two ratios are really equal.
* Percents can be considered as a special kind of ratio, or fraction, or decimal. As a ratio, percents always compares with a number 100 As a fraction, percents always have the denominator of 100. As a decimal, percents have the place value of "hundredths."
* Percents, decimals, and fractions are all interchangeable. There are six possible ways of interchange among them.
* There are three basic types of percent problems because a percent problem is made up of only three quantities: percentage, total, and part.
* In solving percent problems, read and analyze the problem first. Use the information given to write a mathematical equation. Then the equation is rearranged so that the unknown term is alone on the left side of the equation and the known terms on its right side. Any number with the percent sign (%) should be changed to a decimal, or a fraction before it is used in computation.

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Ruth C. Sun
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