the family MATH companion

ARITHMETIC—THE FOUNDATION OF MATH

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Part III. Decimal Operations

A. Introduction

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Decimal Point & Decimal Numbers (Review "Place Value Of Decimals" p.50)

Decimal Point ↓

The Decimal point *separates* the whole number part from the fractional part (See next page.)

a) .25
.250
.2500

Decimals are numbers that are ***less than 1.*** They have digits **only to the right** of decimal point. These digits are called ***decimal digits.***
Note: .25 = .250 = .2500 = .25000. (p.67)

b) 12
12.0
12.00
12.000

Whole Numbers are numbers that ***equal to*** or ***greater than 1.*** They have digits **only to the left** of the decimal point. In whole numbers the decimal point is usually omitted. (See p.228)
Note: 12 = 12.0 = 12.00 = 12.000.

c) 6.85
6.850

Mixed Decimals are numbers that have digits on **both sides** of the decimal point. They are **made up of:**
(Whole Number) + (Decimal)

Remember: Zeros may be added (or dropped) ***at the end*** of a decimal ***without*** **changing its value.**

Decimal Digits Or Fractional Part (See also "Decimals & Decimal Fractions" p.98)

Keep in mind that **decimals** and **fractions** are ***two different ways*** of writing the same number. For example, we can call ".526" the decimal digits or the fractional part of the number 13.526 as seen below:

.526 ⟵ **Decimal digits** or **Fractional part** of 13.526

13.526 ⟵ **A mixed decimal** (= **whole number** + **decimal**)

$13\ \frac{526}{1000}$ ⟵ **A mixed numbers** (= **whole number** + **fraction**)

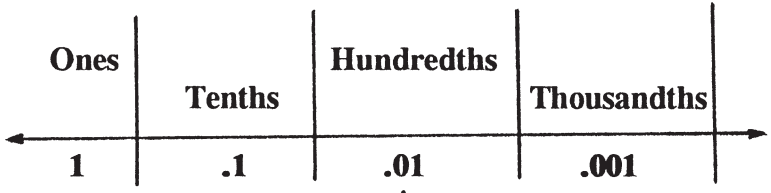
Decimal Digits vs. Digits (See also p.56)

* **Decimal digits** - The words are used with ***decimal numbers.***
  Example: We say "**12.*4518***" has **4 decimal digits.**"

* **Digits** - The word is used with ***whole numbers.***
  Example: We say "**2743 is a 4-digit number.**"

Note: It is important that you learn to read and write decimals correctly. Read page 58.

Borrowing & Carrying With Decimal Numbers



Read first "Decimal System" "Carrying" & "Borrowing" (See pp.140, 158)

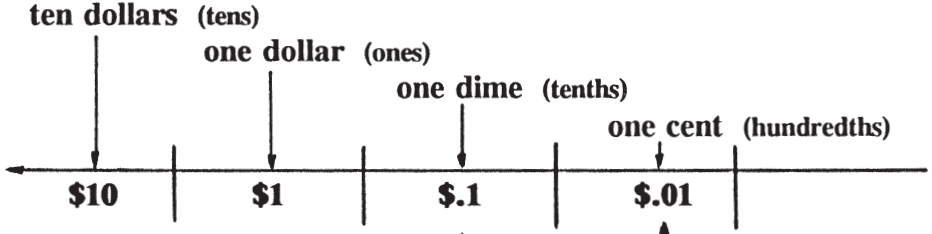
One hundredth (.01) is **10 times larger** than **one thousandth** (.001), the digit to its right.
* ***Borrowing*** **1** hundredth, you get **10** one thousandths.
* ***Carrying*** **10** one thousandths over becomes **1** hundredth.

One tenth (.1) is **10 time larger** than **one hundredth** (.01), the digit to its right.
* ***Borrowing*** **1** tenth, you get **10** one hundredths (.01).
* ***Carrying*** **10** one hundredths over becomes **1** ten.

One (1) is **10 times larger** than **one tenth** (.01), the digit to its right.
* ***Borrowing*** **1**, you get **10** one-tenths (.1).
* ***Carrying*** **10** one tenths over becomes **1**.

Decimal System & U.S. Money System (See also "Decimal & Money" p.59)

Our money system *works exactly* like decimal system, except it uses only two decimal places - the **tenths (dime)** & the **hundredths (cent).**

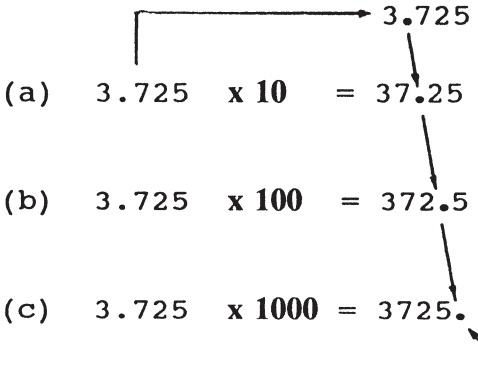


One cent is one tenth (1/10) of one dime.
1 dime equals 10 one-cent.
One cent is **one hundredth** of one dollar.
1 dollar equals 100 one-cent.

Note: $1 = $1.00; $.1 = $.10

One dime is **10 times** more than one cent.
10 one-cent can be exchanged for 1 dime.
One dime is **one tenth (1/10)** of one dollar.
10 dimes can be exchanged for 1 dollar

Multiplying Decimals by 10, 100, etc. (See also p.68)



In writing whole numbers, we omit the decimal point.

Think!

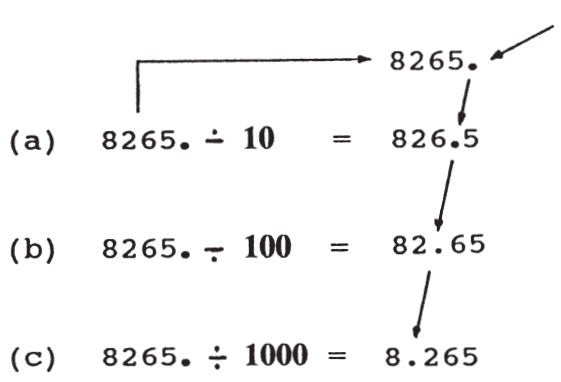
1 Do you divide or multiply when you move the decimal point to the right?
2 Does the number become smaller or larger when the decimal point is moved to the right?
3 Are the number of zeros in the multiplier related to the number of places the decimal point is moved?

General Rule:

When multiplying by 10, 100, etc., move the decimal point *to the right* the same number of places as there are zeros in the multiplier.

Example:
* Multiplying by 10 - **1 zero** - move **1 place.**
* Multiplying by 100 - **2 zeros** - move **2 places.**
* Multiplying by 1000 - **3 zeros** - move **3 places.**

Dividing Decimals By 10, 100, etc. (See also p 69)



Think!

1 Do you divide or multiply when you move the decimal point to the left?
2. Does the number become larger or smaller when the decimal point is moved to the left?
3 Are the number of zeros in the divisor related to the number of places the decimal point is moved?

General Rule:

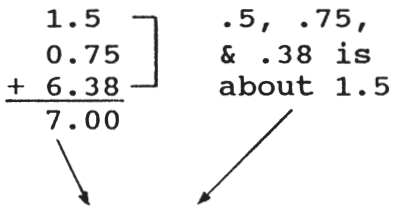
When dividing by 10, 100, etc., move the decimal point *to the left* as many places as there are zeros in the divisor.

Example:
* Dividing by 10 - **1 zero** - move **1 place.**
* Dividing by 100 - **2 zeros** - move **2 places.**
* Dividing by 1000 - **3 zeros** - move **3 places.**

Estimating Sums of Decimals & Money (Review "Rounding" p 30)

We estimate the sums/differences of decimals in the same way we estimate the sums/differences of whole numbers (See p.116)

a) Estimate 1.5 + 0.75 + 6.18.



Method 1. Using Front-end digits & Adjusting
Write the addends vertically with the place values lined up correctly.
1st. Add the whole numbers part: 7.
2nd. Estimate the sum of the decimal part and add to the initial estimate.
8.5 is the adjusted estimate.

b) Estimate $16.58 + $7.89.

Front digits	**Rounding**
$16.58	$17.00
+ 7.89	+ 8.00
$23.00	$25.00

Method 1. Using Front-end digits
Add the whole numbers without adjustment.
Method 2. Using Rounding
Rounding to the nearest whole number, ones digit, **(same place)** and add.

The actual sum is more than $23.00 and less than $25.00.

Estimating Differences of Decimals & Money (Review first p.117)

Remember there are many ways to estimate. Choose a method that is the easiest for the situation and also gives a closer estimation. Think!

a) Estimate 21.75 - 2.04 (**Clue:** Think of 22 - 2.)

21.75	22.00	(up)
- 2.04	- 2.00	(down)
	20.00	

Rounding is a better method. Remember that you round the minuend up and subtrahend down, so the actual difference should be less than 20.

b) Estimate $75.83 - $25.05 (**Clue:** Both have 5 in the ones digit.)

Front Digits	**Rounding**
$75.83	$80.00
- 25.05	- 30.00
$50.00	$50.00

Front Digits (left). Subtract the whole number part.
Rounding (right). Round the numbers to the largest place, to the nearest tens, and subtract. Both methods give the same estimate.

Since .83 > .05, the actual difference is more than $50.

The advantage of using front digits is that we can compare the decimal digits and adjust the estimate.

Estimating Products of Decimals & Money (Review first p.118)

The method is the same as estimating products of whole numbers The key is changing one factor or both to one-digit-number(s) with the remaining digits zeros so that we can multiply mentally.

a) Estimate 28.56 x 2.79

28.56	up	30.00
x 2.79	up	x 3.00
		90.00

Round each factor to **its own** largest place and multiply. Since both factors were rounded up, **adjustment is needed.** The actual product **is less than** 90.

b) Estimate 0.49 x 5.18

0.49	up	0.5
x 5.18	down	x 5
		2.5

Round 0.49 up and round 5.18 down and multiply. No adjustment is needed. It is a reasonable estimate.

c) Estimate $1.49 x 35 (Change both to next whole number.)

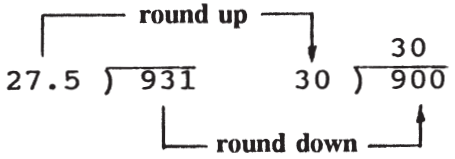
$1.49	up	$2.00
x 35	up	x 40
		$80.00

It is better to overestimate if you are estimating to see whether you have enough money to make a purchase.

Estimating Quotients Of Decimals & Money

In estimating decimal quotients, we use the **multiplication/division facts** to find the compatible numbers to replace the divisor or the dividend, or both, so that we can compute mentally.

a) Estimate 931 ÷ 27.5



* **Dividing Whole Numbers by Decimals:**
(**Clue:** 9 ÷ 3 = 3 and 90 ÷ 30 = 3)
Round up the divisor and round down the dividend. The actual quotient should be larger than the estimate. Do you know why?

b) Estimate $1.98 ÷ 3

$$3 \overline{)\ \$1.98} \longrightarrow 3 \overline{)\ \$1.80} \quad \$.60$$

$$3 \overline{)\ \$1.98} \longrightarrow 3 \overline{)\ \$2.10} \quad \$.70$$

* **Dividing Decimals by Whole Numbers:**
(**Clue:** 1.8 ÷ 3 or 2.1 ÷ 3)
Change the dividend to a **lower number** $1.80. The actual quotient should be larger than $.60.

Change the dividend to a **higher number** $2.10. The actual quotient should be smaller than $.70.

Summary
(Introduction)

* The decimals are numbers that are between 0 and 1 Decimals have digits only to the right of the decimal point while whole numbers have digits only to the left of the decimal point.
* The mixed decimals are made up of whole numbers and decimals. They have digits on both sides of the decimal points.
* The digits to the right of the decimal point are called the decimal digits or the fractional part of a number
* Carrying and borrowing with decimal numbers follows the same rules as that of whole numbers.
* To multiply a decimal by the powers of 10, we move the decimal point to the right the same number of places as there are zeros in the multiplier
* To divide a decimal by the powers of 10, we move the decimal point to the left the same number of places as there are zeros in the divisor
* Make it a habit to estimate the answer before computation. Then check your answer against the original estimation

Part III. Decimal Operations

B. Addition
C. Subtraction
D. Multiplication

Part III. Decimal Operations - Addition

Part III. Decimal Operations - Subtraction

Part III. Decimal Operations - Multiplication

Adding Decimal Numbers (Review first "Addition of Whole Numbers" p.143)

General Rule: * First write the numbers vertically **with the decimal points lined up** in a column. * **Add zeros** if necessary, so that all addends will have same number of the decimal digits. * Then **add the decimals just as with whole numbers.** * Finally, bring the decimal point **straight down to the sum.**

Example: Add 74.3 + 5.168

75 + 5 = 80

Step 1. Estimate. The sum is about 80.

```
  74.3
+ 5.168
```

Step 2. Write the addends under each other with the digits of the **same place value** lined up. When you do, the decimal points lined up also.

```
  74.300
+ 5.168
```

Step 3. To avoid error, add two zeros to the first addend after the decimal digit 3, so that both addends will have the **same number of the decimal digits.**

```
  74.300
+ 5.168
  79.468
```

Step 4. Add the decimal numbers just as you would with whole numbers. Then bring the decimal point straight down the column into the sum.

Remember: Adding zeros *after* a decimal number *does not change* its value.

Adding Whole Numbers & Decimal Numbers (Review "Carrying..." p.140)

Example: Add .427 + 7.61 + 45

45 ⟶ 45.

Step 1. Put a decimal point to the right of the whole number - at the right of the ones digit.

```
   .427
  7.61
+ 45.
------
```

Step 2. Write the addends vertically with the digits lined up correctly or with the decimal points lined up under each other.

```
   .427
  7.610
+ 45.000
--------
```

Step 3. To avoid error, add zeros to addends so that each addend has the same number of decimal digits or places - 3 decimal places.

```
    1
   .427
  7 610
+ 45.000
--------
  53.037
```

Step 4. Add as you would with whole numbers with carrying in two places: from tenths to ones, and again from ones to tens. Then, bring the decimal point straight down to the sum.
Remember: Carrying is always from right to left.

Remember: The **properties, rules,** and **shortcuts,** that apply to the addition of whole numbers, are also **applicable** to the addition of decimal numbers.

Subtracting Decimal Numbers (Review first "Borrowing .. " p.158)

General Rule: * First write the numbers vertically with **the decimal points lined up.** * **Add zeros** if necessary so that each number has the same number of the decimal digits. * Then subtract decimals just **as with whole numbers.** * Finally, bring the decimal point **straight down to the difference.**

Example: Subtract 5.38 - 0.921

```
  5.38
- 0.921
```

Step 1. Write the numbers vertically with the decimal points lined up. **The Larger number always at the top.**

```
  5.380
- 0.921
```

Step 2. Add zero if necessary, so that both numbers have the same number of decimal digits.

```
    7 10
  5.3 80
- 0.921
  4.459
```

Step 3. Subtract as if they were whole numbers.

* Subtract the thousandths: 0 - 1 = impossible.
  Borrow 1 hundredth that makes 10 thousandths. 10 - 1 = 9.
* Subtract the hundredths: 7 - 2 = 5.
* Subtract the Tenths: 3 - 9 = impossible.
  Borrow 1 from the ones that makes 10 tenths. 13 - 9 = 4.
* Subtract the ones: 4 - 0 = 4.
* **Bring the decimal point straight down to the difference.**

Note: The difference should have the same number of decimal digits as those of the largest number

Subtracting Money (See also "Decimals & Money" p.59)

Except for the dollar sign, we subtract (or add) money **in exactly the same way** as we subtract (or add) decimal numbers. Decimals can have many decimal digits, but money uses **only two decimal places: the dime and the cent.**

Example: Subtract $10 - $3.24.

$10 ⟶ $10.00

Step 1. Put a decimal point to the right of the whole number **followed by two zeros.**

```
  $10.00
- $3.24
```

Step 2. Write the number vertically with the decimal points lined up.

```
    9 9 10
  $1̸0.00
- $3.24
  $6.76
```

Step 3. To subtract, we have to borrow across the zeros - the dime & the dollar places. **Replace the 10 dollars with the following:**

9 dollar, 9 dimes, and 10 cents.

Then begin at the right and subtract.

* Subtract the cents (hundredths): 10 - 4 = 6.
* Subtract the dimes (tenths): 9 - 2 = 7.
* Subtract the dollars (ones): 9 - 3 = 6.

The difference is $6.76.

Check:

```
  $6.76
+ $3.24
 $10.00
```

Multiplying A Decimal By A Whole Number

General Rule: First, multiply decimals just as with whole numbers ignoring the decimal points. Then, count the total number of the decimal places in both the multiplicand and the multiplier combined. Finally, point off from the right in the product *the same number of decimal places.*

Example: Multiply 9.37 x 4

```
  1 2
  9.37
x   4
 3748
```

Step 1. Multiply 937 x 4 (Multiply as whole numbers)

1st Multiply 4 x 7 = 28.
Write 8 under 4. Carry 2.

2nd. Multiply 4 x 3 = 12. 12 + 2 = 14.
Write 4 in the same column with 3. Carry 1.

3rd. Multiply 4 x 9 = 36. 36 + 1 = 37.
Write 37 to the left of 4.

```
  9.37   (2 places)
x   4
 37.48   (2 places)
```

Step 2. Put a Decimal Point in the product:

1st. Count the ***total number*** of decimal places in both factors combined: 2 places.

2nd. Point off ***from the right*** 2 places in the product -- 37.48.

Note: In multiplying decimals, you don't think of the decimal points until you come to the product.

Multiplying A Decimal By A Decimal (Review "General Process..." p.182)

```
  .28
x .34
  112 ——— 28 x 4
+ 84  ——— 28 x 3
  952
```

Step 1. Multiply 28 x 4.

1st. Multiply 4 x 8 = 32.
Write 2 under 4. Carry 3.

2nd. Multiply 4 x 2 = 8. 8 + 3 = 11.
Write 11 on the left of 2.

Step 2. Multiply 28 x 3.

1st Multiply 3 x 8 = 24.
Write 4 **in the same column** with 3. Carry 2.

2nd. Multiply 3 x 2 = 6. 6 + 2 = 8.
Write 8 on the left of 4.

```
  .28    (2 places)
x .34    (2 places)
  112
+ 84
 .0952   (4 places)
```

Step 3. Add the partial products: 952

Step 4. Place the Decimal Point in the Product.

1st. Count the **number of decimal places** in both factors combined: **2 + 2 = 4 places.**

2nd. ***Place a zero before 9*** and a **decimal point** before the zero to make **4 places** in the product.

If not enough decimal places in the product, insert zeros between the decimal point and the first decimal digit.

Summary
(Addition, Subtraction, Multiplication)

* Adding zero(s) after a decimal number does not change the value of the number But inserting zero(s) between the decimal point and the first decimal digit does change the value of the decimal - the decimal becomes smaller
* To add or subtract decimal numbers, write the numbers one under the other with the digits lined up according to their place value. If you do, the decimal points will also be lined up.
* To avoid error, add zeros to the empty places so that every number has the same number of decimal digits. Then add or subtract as you would with whole numbers. At the end, bring the decimal point straight down to the sum or the difference.
* To multiply decimals, you don't have to line up the decimal points. Write the decimals vertically and multiply them as if they were whole numbers. Then count the total number of decimal places in both factors combined. Point off from the right the same number of places in the product. If not enough decimal places in the product, add zeros between the decimal point and the first decimal digit.

Part III. Decimal Operations

E. Division

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Dividing Whole Numbers - Mixed Decimals (Review "Division" p.96)

You know a remainder appears when the numbers *can not be divided evenly.* **By adding a decimal point and zero(s) to the dividend, you can continue to divide.** And the quotient will be a mixed decimal. **Example:** Divide 34 ÷ 5.

```
    6
5 ) 34
  - 30
    4
```

Steps 1. Divide: 34 - 5 = 6 r4 Write 6 above 4.
2. Multiply: 5 x 6 = 30. Write 30 under 34.
3. Subtract: 34 - 30 = 4. **4 is smaller than 5.**

To write the remainder as a decimal:
*** Add a decimal point and a zero to the dividend.**
*** Put a decimal point in the quotient right above the one in the dividend. Then continue to divide.**

```
    6.8
5 ) 34.0
  - 30
    4 0
  - 4 0
      0
```

Steps 1. Bring down 0. **40 is the 2nd partial dividend.**
2. Divide: 40 ÷ 5 = 8. Write 8 above 0.
3. Multiply: 5 x 8 = 40. Write 40 under 40.
4. Subtract: 40 - 40 = 0 Write 0 under 0.

Rule: The number of decimal places in the quotient *should equal* the number of decimal places in the dividend.

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Dividing Whole Numbers - Mixed Decimals (Review "Division" p.96)

You know a remainder appears when the numbers *can not be divided evenly.* **By adding a decimal point and zero(s) to the dividend, you can continue to divide.** And the quotient will be a mixed decimal. **Example:** Divide 34 ÷ 5.

```
    6
5 ) 34
  - 30
    4
```

Steps **1.** Divide: 34 - 5 = 6 r4 Write 6 above 4.
2. Multiply: 5 x 6 = 30. Write 30 under 34.
3. Subtract: 34 - 30 = 4. **4 is smaller than 5.**

To write the remainder as a decimal:
*** Add a decimal point and a zero to the dividend.**
*** Put a decimal point in the quotient right above the one in the dividend. Then continue to divide.**

```
    6.8
5 ) 34.0
  - 30
    4 0
  - 4 0
      0
```

Steps **1.** Bring down 0. **40 is the 2nd partial dividend.**
2. Divide: 40 ÷ 5 = 8. Write 8 above 0.
3. Multiply: 5 x 8 = 40. Write 40 under 40.
4. Subtract: 40 - 40 = 0 Write 0 under 0.

Rule: The number of decimal places in the quotient *should equal* the number of decimal places in the dividend.

Dividing Smaller Numbers By Larger Numbers - Terminating Decimals

Remember, when the dividend is *smaller than* the divisor, like the proper fractions, the quotient will be **a decimal, less than 1.**

Example: Divide 1 ÷ 4. (To change 1/4 to a decimal, we divide 1 - 4.)

```
      0.          1st. 1 ÷ 4 = impossible.    Write 0. in quotient above 1.
4 ) 1.0               Add a decimal point and a zero to the dividend.

      0.2         2nd. Divide: 10 ÷ 4 = 2 r2  Write 2 in quotient above 0.
4 ) 1.0               (Treat the new dividend, 1.0, as whole number 10.)
    - 8               Multiply: 4 x 2 = 8.    Write 8 under 0.
      2               Subtract: 10 - 8 = 2.   2 is smaller than 4.

      0.25        3rd. Add another zero to the dividend.
4 ) 1.00              Bring the zero down.    20 is the 2nd partial dividend.
    - 8               Divide 20 ÷ 4 = 5.      Write 5 in the quotient.
      20              Multiply 4 x 5 = 20.    Write 20 under 20.
    - 20              Subtract: 20 - 20 = 0.
      0
```

Rule: Whenever the dividend, remainder included, *is smaller than* the divisor, **add zero(s)** to the dividend or remainder and continue to divide. **Repeat the process until it reaches a "0" remainder** for terminating decimals.

Dividing Smaller Numbers By Larger Numbers - Repeating Decimals

Repeating decimals may occur whether the dividend *is larger*, or *is smaller than* the divisor. Example: Divide 1 - 3. (The fraction 1/3 means 1 - 3.)

```
    0.
3 ) 1.0
```

Steps 1. 1 – 3 = impossible. Write 0. above 1.
2. Add a decimal point and zero to the dividend.

```
    0.3
3 ) 1.0
  - 9
    1
```

Steps 1. Divide: 10 - 3 = 3 r1 Write 3 above 0.
2. Multiply: 3 x 3 = 9. Write 9 under 0.
3. Subtract: 10 - 9 = 1. Write 1 under 9.

```
    0.33
3 ) 1.00
  - 9
    10
   - 9
     1
```

Steps 1. Add another 0 to the dividend.
2. Bring down zero. **10 is the 2nd partial dividend.**
3. Divide: 10 ÷ 3 = 3 r1 Write 3 above 0.
4. Multiply: 3 x 3 = 9. Write 9 under 0.
5. Subtract: 10 - 9 = 1. Write 1 under 9.

You can predict that **3 in the quotient will repeat**, because no matter how far we carry out the division, there will be **a remainder 1.** In such case, we simply **write a bar above the digit**, or the block of the digits, **that repeat**. See the example on the left.

$1 \div 3 = 0.3\overline{3}$

1 ÷ 3 = 0.33...

Connection Among Whole Numbers, Mixed Decimals, And Mixed Numbers

In studying math, it is important to see how the different parts are connected. For example, whole numbers, mixed decimals (decimals), and mixed numbers (fractions) are studied under different subjects, but they may all appear in the **division of whole numbers as seen below.** When you divide whole numbers, you will notice that the quotient could be·

(a) **A Whole Number** - If the dividend *is larger than* the divisor and if it *divides evenly* with 0 as the remainder.
Example: 27 - 3 = 9.

(b) **A Mixed Decimal** (A decimal) - If the dividend *is larger than* the divisor and if it *doesn't divide evenly* The quotient may be **a terminating decimal.**
Example: 17 ÷ 5 = 3.4

(c) **A Mixed Number** (A fraction) - If the dividend *is larger than* the divisor and if it *doesn't divide evenly.* The quotient may be **a repeating decimal.**
Example: 34 ÷ 7 = 4 6/7

The best way to present a repeating decimal is to write it as a mixed number.

Dividing Decimals By Whole Numbers

The following two examples show you *where* to place the decimal points when the dividends are decimals and the divisors are whole numbers.

Example: Divide .21 ÷ 6.

Since 2 can not be divided by 6, write 0 above 2. Remember that 0s after the the decimal point and before the decimal digit are "place holders." They can not be omitted! You see .3 is 10 times larger than .03.

```
    .0 3
6 ) .2 1
  - 1 8
    ---
      3
```

Steps

1. **Put a decimal point in the quotient above the decimal point in the dividend, and followed by 0.**
2. Divide: 21 - 6 = 3 r3 Write 3 above 1.
3. Multiply: 6 x 3 = 18.
4. Subtract: 21 - 18 = 3. 3 is smaller than 6.

```
    .0 3 5
6 ) .2 1 0
  - 1 8 |
    -----
      3 0
    - 3 0
      ---
        0
```

Steps

1. **Add a zero to the dividend.**
2. Bring down 0. **30 is the 2nd partial dividend.**
3. Divide: 30 - 6 = 5. Write 5 above 0.
4. Multiply: 6 x 5 = 30. Write 30 under 30.
5. Subtract: 30 - 30 = 0.

Example: Divide 35.07 ÷ 7 = 5.01

To the left of the decimal point are whole numbers. And 0s before whole numbers are dropped since they have no value: 05 = 5 (See p.66)

```
    05.
7 ) 35.07
  - 35
    0
```

Steps
1. Divide: 3 ÷ 7 = impossible.
2. Divide: 35 ÷ 7 = 5. Write 0 above 3.
3. Multiply: 7 x 5 = 35. Write 5 above 5.
4. Subtract: 35 - 35 = 0. Write 35 under 35. Write 0 under 5.

```
    5.01
7 ) 35.07
  - 35 ↓↓
    0 07
    - 7
      0
```

Steps
1. **Put a decimal point after 5.**
2. Bring down 0. **Write 0 in the quotient above 0.** (Since 0 ÷ 7 = 0)
3. Bring down 7. **7 is the 2nd partial dividend.**
4. Divide: 7 - 7 = 1. Write 1 above 7.
5. Multiply: 7 x 1 = 7. Write 7 under 7.
6. Subtract: 7 - 7 = 0.

Note: A mixed decimal (35.07) = whole number (35) + decimal (.07)

35.07 ÷ 7 = (35 ÷ 7) + (.07 ÷ 7) = 5 + .01 = 5.01

whole number **decimal** **quotient**

Changing Decimal Divisors To Whole Numbers (Review first p.228)

When the divisor is a decimal, **you must make it a whole number** before you can carry out the division. The following shows the how-to:

/04) /26. → 4) 26

Move the decimal point 2 places to the right:
* The divisor: .04 — (x 100) → 4 (Drop 0)
* The dividend: .26 — (x 100) → 26

/22) 8/8. → 22) 880

Move the decimal point 2 places to the right:
* The divisor: .22 — (x 100) → 22
* The dividend: 8.8 — (x 100) → 880 (Add 0)

1/3) /39 → 13) 3.9

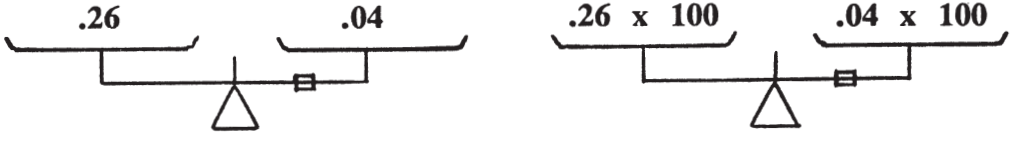
Move the decimal point 1 place to the right:
* The divisor: 1.3 — (x 10) → 13
* The dividend: .39 — (x 10) → 3.9

/6) /0018 → 6) .018

Move the decimal point 1 place to the right:
* The divisor: .6 — (x 10) → 6
* The dividend: .0018 — (x 10) → .018

Remember: To move the decimal point to the right is the same as multiplying the number by the powers of 10: 10, 100, etc.

Error To Avoid: In changing decimal divisors to whole numbers, students tend to change the divisor without changing the corresponding dividend. Keep in mind that the relation between the divisor and the dividend is like a balance.



In order to maintain the balance, you must do to the dividend what you have done to the divisor.

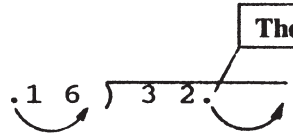
Another way to avoid the error is to write the division in fraction form:

$$\frac{\text{dividend}}{\text{divisor}} \quad \frac{.26}{.04} = \frac{.26 \times 100}{.04 \times 100} = \frac{26}{4}$$

If you multiply the divisor by 100, you must also multiply the dividend by 100. 26/4 is an equivalent fraction of .26/.04. (See p.301)

Dividing Whole Numbers By Decimals

Read first the previous two pages before you study the following example. **Example:** Divide 32 - .16



1st. Make *the divisor* a whole number **by moving the decimal point to the right of 6** - 2 places.

```
        2 0 0
1 6 ) 3 2 0 0
    - 3 2
      0 0
```

2nd. Move the decimal point of *the dividend* **an equal number of places** - 2 places. **And fill the vacant places with zeros.** Then divide.

Again, we use *fractions* to show the steps given above:

$$\frac{32}{.16} = \frac{32 \times 100}{.16 \times 100} = \frac{3200}{16}$$

32/.16 and 3200/16 **are equivalent fractions** with *different numbers* only.

When we move the decimal point of the divisor *2 places to the right,* we **multiplied the bottom number by 100.** In order to **keep the value** of the original fraction, we **must also** multiply the top number (the dividend) by 100.

Example: Divide 752 - .8 (The division in fraction is 752/.8)

$8\overline{)752.} = 8\overline{)7520}$

First. Make the divisor a whole number by moving the decimal points of ***both*** **the divisor and dividend 1 place to the right.** Then divide.

```
      9 4 0
   ---------
8 ) 7 5 2 0
  - 7 2   ↓
  -------  |
      3 2  |
    - 3 2  ↓
    -------
        0 0
      - 0 0
      -----
          0
```

Steps 1. Divide 75 ÷ 8 = 9 r3 Write 9 above 5.
2. Multiply 8 x 9 = 72. Write 72 under 75.
3. Subtract 75 - 72 = 3. Write 3 under 2.
4. Bring down 2. Write next to 3.

Steps 1. Divide 32 - 8 = 4. Write 4 above 2.
2. Multiply 8 x 4 = 32. Write 32 under 32.
3. Subtract 32 - 32 = 0. Write 0 under 2.
4. Since 0 ÷ 8 = 0. Write 0 above 0.

Check: Is 752 - .8 = 940? We know **"quotient x divisor = dividend".**

```
  Quotient        940
x  Divisor      x  .8   (1 decimal place)
  Dividend      752.0   (1 decimal place)     752.0 = 752
```

Note: 752 ÷ .8 is 940 and 7520 ÷ 8 is also 940.

Dividing Decimals By Decimals (Review first p.228)

Example: Divide .0021 ÷ .6

.6) .0021 = 6) .021

Move the decimal points of the divisor and the dividend 1 place to the right.

0s to the right of the decimal point, and before the decimal digits are "place-holders." They can not be omitted.

```
   .003
6 ).021
  - 18
    3
```

Steps
1. **Put a decimal point in the quotient above the decimal point in the dividend.**
2. Divide 0 ÷ 6 = 0. Write 0 above 0.
3. Divide 2 ÷ 6 = impossible. Write 0 above 2.
4. Divide 21 - 6 = 3 r3. Write 3 above 1.
5. Multiply 6 x 3 = 18. Write 18 under 21.
6. Subtract 21 - 18 = 3. Write 3 under 8.

```
   .0035
6 ).0210
  - 18
    30
  - 30
     0
```

Steps
1. Add 0 to the dividend.
2. Bring down 0. Write 0 next to 3.
3. Divide 30 - 6 = 5. Write 5 above 0.
4. Multiply 6 x 5 = 30. Write 30 under 30.
5. Subtract 30 - 30 = 0. Write 0 under 0.

Example: Divide 49.247 ÷ 1.21

1.21) 49.247 = 121) 4924.7 Move two places to the right.

Since 4 or 49 can not be divided by 121, we are supposed to put 0s above those two digits. But we drop them because 0s before whole numbers have no value.

```
      004
121 ) 4924.7
    - 484
      84
```

Steps
1. Divide 492 ÷ 121 = 4 r8. Write 4 above 2.
2. Multiply 121 x 4 = 484. Write 484 under 492.
3. Subtract 492 - 484 = 8. Write 8 under 4.
4. Bring down 4. Write 4 next to 8.

```
      40.7
121 ) 4924.7
    - 484
      84 7
      84 7
       0
```

Steps
1. 84 can't be divided by 121. Write 0 above 4.
2. **Put a decimal point in the quotient right above the decimal point in the dividend.** Bring down 7.
3. Divide 847 ÷ 121 = 7. Write 7 above 7.
4. Multiply 121 x 7 = 847.
5. Subtract 847 - 847 = 0.

Remember: The decimal places of the quotient **must equal** the decimal places of the dividend.

Dividing Money

Dividing money is done *exactly the same* as dividing with decimals or whole numbers. Example: Divide $18.00 ÷ 24

Since 0s means no dollars, we drop them but keep the dollar sign and the cent point (decimal point).

```
      $ØØ.7
24 ) $18.00
     - 16 8
       1 20

      $  .75
24 ) $18.00
     - 16 8
       1 20
     - 1 20
         0
```

Steps 1. 1 or 18 can't be divided by 24, **write 0s above 1 and 8, and followed by a decimal (or cent) point.**
2. Divide 180 ÷ 24 = 7 r12. **Write 7 in dime place.**
Think: 25 x 4 = 100 & 25 x 8 = 200.
3. Multiply 24 x 7 = 168. Write 168 under 180.
4. Subtract 180 - 168 = 12. Write 12 under 68.
5. Bring down 0. Write next to 2.

Steps 1. Divide 120 ÷ 24 = 5. **Write 5 in cent place.**
2. Multiply 24 x 5 = 120 Write 120 under 120.
3. Subtract 120 - 120 = 0. Write 0 under 0.

The answer is **$.75.**

Note: * The **cent (decimal) point** is located **two places from the right.**
* Whole dollars can be written as either $18 or $18.00.

Dividing Money With Repeating Decimals

When dividing with decimals, some quotients appear to continue without end - they are repeating decimals. We use rounding to handle such cases in the following way:

First, Determine an appropriate **number of decimal places** for the quotient.
Then, Divide to **one additional place** - carry the division one extra place.
Finally, Round the quotient to the desired (or required) place.

In general, money is rounded to the nearest cent - 2 decimal places.

Example: Divide to the nearest cent: $17 ÷ 3.

```
        $5.666
    ↓
3 ) $17.000
   - 15   ↓ ↓ ↓
     2 0  ↓ ↓ ↓
   - 1 8  ↓ ↓ ↓
       20 ↓ ↓ ↓
     - 18 ↓ ↓ ↓
        20  ↓ ↓
      - 18  ↓ ↓
         2  ↓ ↓
```

1st. Divide. Keep the dollar sign ($) to serve as a reminder.

2nd. **Mark the hundredths (cent) place** and carry the division ***one extra place*** - to the thousandths place.

3rd. **Round** the quotient **to the nearest hundredth (cent)** place - 2nd place to the right of the decimal point. Since 6 in the thousandths place is greater than 5, **add "1"** to the "cent" place. The answer: $5.67

Summary
(Division)

* In dividing whole numbers, if the divisor is smaller than the dividend, the quotient can be a whole number or a mixed decimals. But if the divisor is larger than the dividend, the quotient will be a decimal - either a terminating decimal or a repeating decimal.
* In division, if the divisor is a decimal, make it a whole number by moving the decimal point to the right of the last digit. Then move the decimal point of the dividend an equal number of places. If the dividend is a whole number, fill the place(s) with zero(s)
* Division with decimal numbers is similar to division with whole numbers. Special attention is directed to the place of decimal points in quotients.
* The decimal point in the quotient should be right above the decimal point in the dividend. And the number of decimal places in the quotient should equal the number of decimal places in the dividend.
* Mixed numbers is the best way to write a repeating decimal. In general, money is rounded to the nearest cents (hundredth place)