the family MATH companion

ARITHMETIC—THE FOUNDATION OF MATH

Ruth C. Sun

Part II. Whole Number Operations

B. Addition

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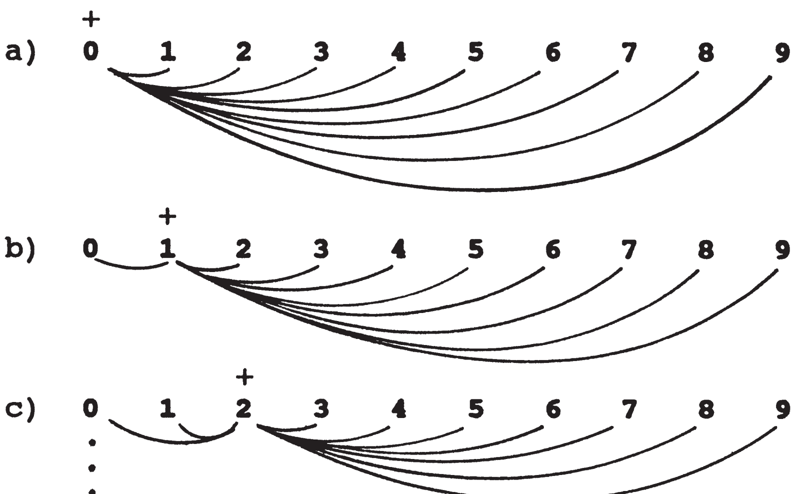
Addition Facts: Combinations of Ten Digits (See "Ten Digits" p.106)

The following shows how we got our addition facts. They are the combinations of the ten digits (0 to 9). Start with 0. Add 0 to itself, then added to each of the remaining digits, one at a time in order, as seen in a). Follow the same procedure for each digit, 1 through 9, and you will have the 100 basic addition facts found on the next page.



0 + 0
0 + 1
0 + 2
0 + 3

1 + 0
1 + 1
1 + 2

2 + 0
2 + 1
2 + 2

Do you see some facts repeat?

0 + 1 = 1 + 0

0 + 2 = 2 + 0

1 + 2 = 2 + 1

Commutative Prop.
(See p.130)

Note: The sign "+" serves to remind you that you always add the number to itself.

100 Basic Addition Facts - One-Digit Addition

0 + 0 = 0	1 + 0 = 1	2 + 0 = 2	3 + 0 = 3	4 + 0 = 4
0 + 1 = 1	1 + 1 = 2	2 + 1 = 3	3 + 1 = 4	4 + 1 = 5
0 + 2 = 2	1 + 2 = 3	2 + 2 = 4	3 + 2 = 5	4 + 2 = 6
0 + 3 = 3	1 + 3 = 4	2 + 3 = 5	3 + 3 = 6	4 + 3 = 7
0 + 4 = 4	1 + 4 = 5	2 + 4 = 6	3 + 4 = 7	4 + 4 = 8
0 + 5 = 5	1 + 5 = 6	2 + 5 = 7	3 + 5 = 8	4 + 5 = 9
0 + 6 = 6	1 + 6 = 7	2 + 6 = 8	3 + 6 = 9	4 + 6 = 10
0 + 7 = 7	1 + 7 = 8	2 + 7 = 9	3 + 7 = 10	4 + 7 = 11
0 + 8 = 8	1 + 8 = 9	2 + 8 = 10	3 + 8 = 11	4 + 8 = 12
0 + 9 = 9	1 + 9 = 10	2 + 9 = 11	3 + 9 = 12	4 + 9 = 13

5 + 0 = 5	6 + 0 = 6	7 + 0 = 7	8 + 0 = 8	9 + 0 = 9
5 + 1 = 6	6 + 1 = 7	7 + 1 = 8	8 + 1 = 9	9 + 1 = 10
5 + 2 = 7	6 + 2 = 8	7 + 2 = 9	8 + 2 = 10	9 + 2 = 11
5 + 3 = 8	6 + 3 = 9	7 + 3 = 10	8 + 3 = 11	9 + 3 = 12
5 + 4 = 9	6 + 4 = 10	7 + 4 = 11	8 + 4 = 12	9 + 4 = 13
5 + 5 = 10	6 + 5 = 11	7 + 5 = 12	8 + 5 = 13	9 + 5 = 14
5 + 6 = 11	6 + 6 = 12	7 + 6 = 13	8 + 6 = 14	9 + 6 = 15
5 + 7 = 12	6 + 7 = 13	7 + 7 = 14	8 + 7 = 15	9 + 7 = 16
5 + 8 = 13	6 + 8 = 14	7 + 8 = 15	8 + 8 = 16	9 + 8 = 17
5 + 9 = 14	6 + 9 = 15	7 + 9 = 16	8 + 9 = 17	9 + 9 = 18

Addition Properties & Addition Facts (Review first "Properties" p.120)

It is **very** important that you memorize the addition facts because all future addition problems and other operations, depend on these basic facts. The following shows that by using **two addition properties,** you can cut down the memory work by half.

0 + 1 = 1	or	1 + 0 = 1
0 + 2 = 2		2 + 0 = 2
0 + 3 = 3		3 + 0 = 3
0 + 4 = 4		4 + 0 = 4
0 + 5 = 5		5 + 0 = 5

The **zero property** of addition says, **"Any number plus 0 is the number."** If you know this property, you can omit 19 facts from the list - they are the ones inside the boxes. (See p.129)

0 + 1	=	1 + 0
3 + 5	=	5 + 3
4 + 7	=	7 + 4
6 + 8	=	8 + 6
9 + 2	=	2 + 9

The **Commutative property** of addition says, **"The order in which numbers are added does not affect the result."** Again, by using this property, we cut down the numbers of facts to be memorized by half.
(See next page.)

Remember: These two addition properties apply also to other addition problems including decimals and fractions.

Rearranging The Addition Facts

By using the **zero property** and the **commutative property** of addition, the numbers of addition facts to be learned are reduced to 45 as listed below. In practicing, **make it a habit** of saying each addition facts **both ways** before giving the sum like this: **3 + 5 = 5 + 3 = 8** or **3 + 5 equals 5 + 3 is 8.**

```
* 1 + 1 = 2
  1 + 2 = 2 + 1 = 3
  1 + 3 = 3 + 1 = 4
  1 + 4 = 4 + 1 = 5
  1 + 5 = 5 + 1 = 6
  1 + 6 = 6 + 1 = 7
  1 + 7 = 7 + 1 = 8
  1 + 8 = 8 + 1 = 9
  1 + 9 = 9 + 1 = 10
* 2 + 2 = 4
  2 + 3 = 3 + 2 = 5
  2 + 4 = 4 + 2 = 6
  2 + 5 = 5 + 2 = 7
  2 + 6 = 6 + 2 = 8
  2 + 7 = 7 + 2 = 9
  2 + 8 = 8 + 2 = 10
  2 + 9 = 9 + 2 = 11
* 3 + 3 = 6
  3 + 4 = 4 + 3 = 7
  3 + 5 = 5 + 3 = 8
  3 + 6 = 6 + 3 = 9
  3 + 7 = 7 + 3 = 10
  3 + 8 = 8 + 3 = 11
  3 + 9 = 9 + 3 = 12
* 4 + 4 = 8
  4 + 5 = 5 + 4 = 9
  4 + 6 = 6 + 4 = 10
  4 + 7 = 7 + 4 = 11
  4 + 8 = 8 + 4 = 12
  4 + 9 = 9 + 4 = 13
* 5 + 5 = 10
  5 + 6 = 6 + 5 = 11
  5 + 7 = 7 + 5 = 12
  5 + 8 = 8 + 5 = 13
  5 + 9 = 9 + 5 = 14
* 6 + 6 = 12
  6 + 7 = 7 + 6 = 13
  6 + 8 = 8 + 6 = 14
  6 + 9 = 9 + 6 = 15
* 7 + 7 = 14
  7 + 8 = 8 + 7 = 15
  7 + 9 = 9 + 7 = 16
* 8 + 8 = 16
  8 + 9 = 9 + 8 = 17
* 9 + 9 = 18
```

Note: The addition facts with * have identical addends.

Addition Table

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

← Addends (top row)

↑ Addends (left column)

Study the table carefully. Do you see the addition properties? Do you see different pairs of numbers that add up to the same number?

Using The Addition Table To Find Sums

In the addition table, the digits at the top and the digits at the left side are addends. There are **two ways** of looking up the sum of two numbers because ***additions are commutative***. See the example below:

				b	a	
	+	0	1	2	3	4
	0	0	1	2	3	4
	1	1	2	3	4	5
a	2	2	3	4	$\boxed{5}$	6
b	3	3	4	$\boxed{5}$	6	7

Finding The Sum: 3 + 2 = 2 + 3 = $\square$

* Find 3 in the top row a , and 2 in the left column a . Then go down the column of 3 and go across the row of 2. **The sum is the number where the column and the row meet.** Or
* Find 3 in the left column b , and 2 in the top row b . **The sum is where the column and the row meet.**

Patterns In Addition Table

Study carefully the addition table and see whether you can find the following:

* Do you see all the addition properties?
* Do you see the different pairs of numbers that give the same sum?

Numbers Adding Up To 10 (See also "Subtracting From 10" p.155)

When adding a column of numbers, **always look for numbers, two or three, that add up to 10.** It helps to speed up the computations. See the examples below:

a) $\begin{array}{r} 4 \\ 3 \\ 6 \\ +\ 7 \\ \hline 20 \end{array}$ b) $\begin{array}{r} 7 \\ 5 \\ 2 \\ +\ 1 \\ \hline 15 \end{array}$ c) $\begin{array}{r} 16 \\ 6 \\ 3 \\ +\ 4 \\ \hline 29 \end{array}$ d) $\begin{array}{r} 1 \\ 8 \\ 9 \\ +\ 2 \\ \hline 20 \end{array}$

In a) 4 + 6 = 10; 3 + 7 = 10
b) 7 + 2 + 1 = 10
c) 16 + 4 = 20
d) 1 + 9 = 10; 8 + 2 = 10
Here we use the associative property

Use the following diagram to help you **memorize** the two numbers and three numbers that add up to ten.

Two numbers add up to 10:

1 2 3 4 5 6 7 8 9

5 + 5
4 + 6
3 + 7
2 + 8
1 + 9

Three numbers add up to 10:

1 + 1 + 8 = 10
1 + 2 + 7 = 10
1 + 3 + 6 = 10
1 + 4 + 5 = 10
2 + 2 + 6 = 10
2 + 3 + 5 = 10
2 + 4 + 4 = 10
3 + 3 + 4 = 10

Applying What You Know

Put your brain to work when studying mathematics! When doing math work, take a good look at the problem first. Always look for ways that you can apply what you already know **to simplify the computation.** For example, if you know the numbers that add up to 10, then you can compute the following problems mentally.

a) If you know: $2 + 8 = 10$
Then apply: $12 + 8 = 20$
$32 + 18 = 50$ ← So, $(32 + 8) + 10 = 50$, or $30 + (2 + 18) = 50$

Remember, you can break numbers apart!
Think: $32 = 30 + 2$; $18 = 10 + 8$.

b) If you know: $2 + 5 = 7$
Then apply: $20 + 50 = 70$
$200 + 500 = 700$

Check:
$$\begin{array}{r} 20 \\ +\ 50 \\ \hline 70 \end{array} \qquad \begin{array}{r} 200 \\ +\ 500 \\ \hline 700 \end{array}$$

If the addends have equal numbers of zeros at the end, add the non-zero digits and keep the number of zeros.

c) If you know: $4 + 6 = 10$
Then apply: $40 + 60 = 100$
$400 + 600 = 1000$

Check:
$$\begin{array}{r} 40 \\ +\ 60 \\ \hline 100 \end{array} \qquad \begin{array}{r} 400 \\ +\ 600 \\ \hline 1000 \end{array}$$

Playing With Numbers (I) - Breaking Numbers Apart

In the process of learning the basic facts, many students tend to see these facts as something fixed. In fact, **any digit or number larger than 1 can be broken apart into a sum of two smaller numbers.** If you know the **rules of mathematics,** you can manipulate numbers with ease. For example:

a) $2 + 5 = 7$. **Remember 7 can also be written as: 0 + 7, 1 + 6, 3 + 4.**

$2 + 5 = 3 + \square$ **Replace 7 with 3 + 4.** Then cover 4 and ask your friend to **find the missing addend.** ***Fun!***

$2 + 5 = 5 + \square$ How about **replace 7 with 5 + 2.** Cover 2 and ask your friend to **name the property!** ***Fun!***

b) $8 + 5 = \square$ You can break either 8 or 5 apart into two numbers, so there will be **two numbers that add up to 10.** ***Speed up!***

$8 + 5 = \square$ **Think:** $8 = 3 + 5$. So, $3 + (5 + 5) = 3 + 10 = 13$

$8 + 5 = \square$ **Think:** $5 = 2 + 3$. So, $(8 + 2) + 3 = 10 + 3 = 13$

Rule: **You always compute the numbers inside the parentheses first.**
So, put the numbers that add to 10 inside the parentheses.

c) $13 - 4 = \square$

Break 13 apart. **subtract 4 from 10.** Then add the difference to the ones digit. **(See p.155)**

$10 + 3 - 4 =$

Think: 13 = 10 + 3.

$(10 - 4) + 3 =$

Rearrange the numbers: Subtract 4 from 10.

$6 + 3 = 9$

Add the difference to 3. *Easy!*

d) $5 \times 27 = \square$

If you know that you can break numbers apart, you can solve the problem in your head.

$5 \times (20 + 7) =$

Think: 27 = **20 + 7,** because 5 x 20 is easy.

$(5 \times 20) + (5 \times 7) =$

Distributive property over addition.

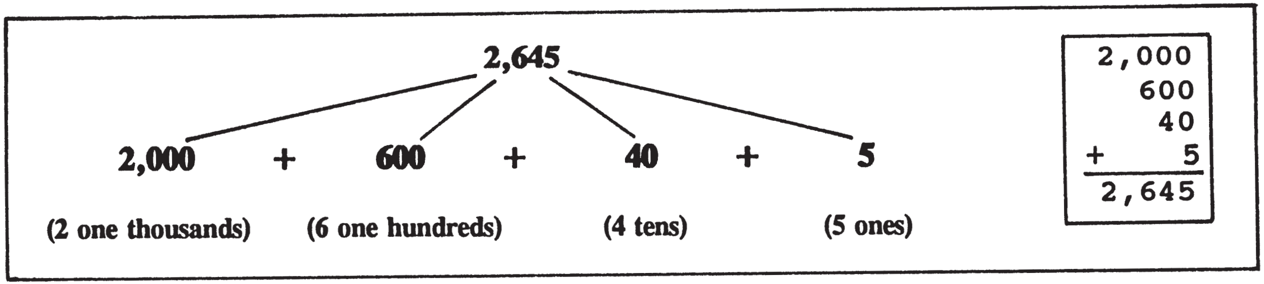
$100 + 35 = 135$

Multiply 5 x 27, you will get 135.

The examples given above are only a sample of what you can do with numbers. If you know numbers can be taken apart, you can speed up computations and turn many problems into **mental math.** ***Math is fun!***

Writing Numbers in Expanded Form (See also "Expanded Form" p.24)

To write a number in expanded form is to write it **as the sum of the value of each digit.** The following shows 2645 in expanded form:



A number can be expressed in various forms. We can write 2645

a) **in words:** **"two thousands, six hundred forty-five"**

b) **in standard numeral:** **2,645 (or 2645)**

d) **in expanded form:** **2,645 = 2,000 + 600 + 40 + 5**

It is important that you learn to read and write numbers correctly, especially large numbers. You can find the instructions on pages 18-19.

Addition Using Expanded Form (Review first the previous page.)

Writing addition problems in expanded form can help us to see the process of addition. First, write each addend in an expanded form.

Example: Add. 85 + 67

$$\begin{array}{rcrl}
85 & = & 80 + 5 & \\
\underline{+\ 62} & = & \underline{60 + 2} & \\
7 & & 140 + 7 & = 147 \\
\underline{+140} & & & \\
147 & & &
\end{array}$$

Writing each addend in expanded form helps us to see, that when we add **8 + 6 = 14,** we are actually adding **80 + 60 = 140** with **0 in the ones place omitted.** Do you know Why?

Example: Add. 8 + 97 + 103

$$\begin{array}{rcrl}
{}^{1\,1} & & & \\
8 & & 8 & \\
97 & = & 90 + 7 & \\
\underline{+103} & = & \underline{100 + 00 + 3} & \\
208 & & 100 + 90 + 18 & = 208
\end{array}$$

Make sure the place value of the digits are **lined up correctly.** Then,

- Add the ones: 8 + 7 + 3 = 18. ***Carry* one 10 to the tens place by adding 1 to the tens.**
- Add the tens: 10 + 90 = 100. ***Carry* one 100 to the hundreds place.**

Carrying In Addition & Decimal System

Hundreds	Tens	Ones
1	1	1
(100)	(10)	(1)

Remember that in decimal system or base 10 system the value of each place is **10 times larger** than the place to its right.

Hundreds	Tens	Ones
0	0	0
1	1	1
2	2	2
.	.	.
.	.	.
.	.	.
9	9	9

According to base 10 system, each place (ones, tens, hundreds, etc.) can have ***only*** **one-digit numbers: 0, 1, 2, 3, ...** ***up to 9.*** That means we ***can not*** **have two-digit numbers** like 10, 11, 12..., etc. in any place. See the example below. Because of this, we have **"carrying"** in addition.

	Hundreds	Tens	Ones
incorrect			~~15~~
correct		1	5

Therefore, in the process of adding numbers, **if the sum of any place (ones, tens, hundreds, etc.) is 10 or more,** we have to ***carry*** the 10, 20, etc. ***to the next higher place on the left.***

(Continued on the next page.)

Examples Of Carrying In Addition (First, read the previous page.)

(a)
$$\begin{array}{r} 4 \\ +\quad 5 \\ \hline 9 \end{array}$$

(b)
$$\begin{array}{r} 2 \\ +\quad 8 \\ \hline 1\ 0 \end{array}$$

(c)
$$\begin{array}{r} 1 \\ 7 \\ +\quad 2\ 4 \\ \hline 3\ 1 \end{array}$$

(d)
$$\begin{array}{r} 2 \\ 9 \\ 6 \\ +\quad 4\ 8 \\ \hline 6\ 3 \end{array}$$

(e)
$$\begin{array}{r} 5\ 5 \\ +\quad 9\ 2 \\ \hline 1\ 4\ 7 \end{array}$$

(a) Adding *ones* place: 4 + 5 = 9. *No carrying.*

(b) Adding *ones* place: 2 + 8 = 10. ***Carrying.***
10 ones = 1 ten. So, write 1 in the tens place & 0 in the ones place.

(c) Adding *ones* place: 7 + 4 = 11. **Carrying.**
11 = 10 + 1 Carry 10 ones to the tens place by adding 1 (10 + 20 = 30)

(d) Adding *ones* place: 9 + 6 + 8 = 23. ***Carrying.***
23 = 20 + 3. Carry 20 ones to the tens place by adding 2. (20 + 40 = 60)

(e) Adding *tens* place: 5 + 9 = 14. ***Carrying.***
14 = 10 + 4. Carry 10 tens to hundreds place by adding 1

Note: We use the word "carrying" only when the place we carry to has numbers like (c) & (d).

Writing Numbers In Vertical Form (Read first "Place Value" p.18)

Math problems are often written horizontally to save space like the following examples:

(a) 2 + 4 + 6 + 8 (b) 17 + 21 + 39 + 13 (c) 15 + 7 + 243

But to compute two- or more digit numbers like (b) and (c) above, you have to rewrite the numbers vertically, one under the other. Since our number system is a place value system, **it is very important that you line up the digits correctly according to their place value.** Let's use (c) as an example.

$$\begin{array}{r} 1\ 5 \\ 7 \\ +\ 2\ 4\ 3 \\ \hline \end{array}$$

Since 15, 7, 243, each has different number of digits, digits must be lined up correctly so that you will **add the digits with the same place value.** For example:

- 5, 7, 3, are lined up in the **ones** column;
- 1, 4, are lined up in the **tens** column;
- 2 alone in the **hundreds** column.

Do you know what would happen, if you **carelessly** write 7 under 1 (tens place) instead of 5 (ones place)? 7 becomes 70, **10 times larger.** keep place value in mind when you write numbers.

General Rules For Adding Numbers

You can compute ***any*** addition problem, large and small, **with confidence, if:**

1. **You have mastered the addition facts.** (See p.131)
2. **You understand the place value concept.** (See p.18)
3. **You have learned the skill of carrying.** (See p.140)

The digits must be lined up correctly.

Remember to ***add*** the number that was carried over.

Adding **always begins at the ones place**, then the tens place,...

```
  '
  58
+ 25
----
  83
```

You can **add down** (8 + 5) or **add up** (5 + 8).

Write the answer under the line. Make sure **each digit is in the right place.**

Always check your answer. If added down, check by adding up or vice versa.

Addition - Carrying More Than 10 (Read first p.140)

The **speed** and **accuracy** with which you add **depends on your knowledge of the basic addition facts.** Remember, accuracy comes first; speed second.

```
   2
  38
  27
+ 59
----
   4
```

Step 1. Add ones' place.

* Add: 8 + 7 + 9 = 24 (24 = 20 + 4).
* Write 4 in the ones place under 9.
  Carry 20 to the tens place by writing 2 above 3.

```
   2
  38
  27
+ 59
----
 124
```

Step 2. Add tens' place.

* Add: *2* + 3 + 2 + 5 = 12 (12 means 100 + 20).
  (2, above 3, is carried over from the ones place.)
* Write 2 in the tens place under 5 and 1 at the left of 2, since there is no hundreds to add to.

To Check, reverse the order (add up):

* Add ones' place: 9 + 7 + 8 = 24 ✓
* Add tens' place: 2 + 3 + 2 + 5 = 12. ✓

Remember: When adding three or more addends, you could carry more than 10 from the ones' place to the tens' place.

Addition - Carrying More Than One Place (Read first p.140)

$$\begin{array}{r} {\scriptstyle 1} \\ 279 \\ +\ 486 \\ \hline 5 \end{array}$$

Step 1. Add ones' place.

* Add: 9 + 6 = 15. (15 = 10 + 5)

* Write 5 in the ones place. *Carry 1 to the tens place.*

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 279 \\ +\ 486 \\ \hline 65 \end{array}$$

Step 2. Add tens' place.

* Add: *1* + 7 + 8 = 16 (16 means 100 + 60)

(1, above 7, was carried over from the ones place.)

* Write 6 in the tens place.

Carry 1 to the hundreds place.

$$\begin{array}{r} {\scriptstyle 1} \\ 279 \\ +\ 486 \\ \hline 765 \end{array}$$

Step 3. Add hundreds' place.

* Add: *1* + 2 + 4 = 7 (7 means 700)

(1, above 2, was carried over from the tens place.)

* Write 7 in the hundreds place under 4.

To Check, reverse the order (add up).

* Add ones' place: 6 + 9 = 15 ✓

* Add tens' place: 8 + 7 + 1 = 16 ✓

* Add hundreds place: 4 + 2 + 1 = 7 ✓

Summary
(Addition)

* Addition facts are used to compute all addition problems. These facts are also used in multiplication. Every math student should memorize them.
* From the addition table, you can find the sums of all one-digit additions, the addition properties, and different pairs of numbers that add up to the same number.
* To simplify computation, look for numbers, two or three, that add up to 10. Use what you already know to speed up your work.
* Writing numbers in expanded form can help us understand the process of adding two- or more digit numbers.
* The decimal system makes "carrying" in addition necessary At any place (ones, tens,), if the sum is 10 or more, carry the 10 to the next higher value place. Sometimes, you may have to carry 20 or more.
* To add two-or more digit numbers, write the numbers in vertical form with the value of each digit lined up correctly Then, start adding the numbers from the ones place, then the tens place, Always in that order Remember to add the numbers that are being carried over. Answer must also be lined up correctly according to their place value.
* Making it a habit of checking your answer If you added down, check by adding up. Addition is commutative.

Part II. Whole Number Operations

C. Subtraction

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Subtraction - An Inverse Operation of Addition (Review first p.110)

Subtraction is the **opposite** of addition. Instead of "adding on," we are "taking away." **For every addition fact, there are two subtraction facts:**

$$\overbrace{5 \quad + \quad 2}^{\text{addends}} \quad = \quad \overset{\text{sum}}{7}$$

$$\overset{\text{sum}}{7} \quad - \quad \overbrace{2 \quad = \quad 5}^{\text{addends}}$$

$$\underset{\text{minuend}}{7} \quad - \quad \underset{\text{subtrahend}}{5} \quad = \quad 2$$

difference

Inverse Operations - The following examples show what we mean when we say **subtraction is an inverse operation of addition** - an operation in reversed order:

$7 - 2 = 5$ (2 + 5 = 7) and $7 - 5 = 2$ (5 + 2 = 7)

Remember: ***Subtractions are not commutative.*** Review the special rule for subtracting whole numbers found on page 122.

Using The Addition Table To Find Differences (See "Addition Table" p.132)

Again, the following example shows how subtraction is related to addition. **To find the difference is the same as to find the missing addend:**

$7 - 3 = \square$ **In subtraction,** we are finding the difference.
$3 + \square = 7$ **In addition,** we are finding the missing addend.

Just as in addition, there are **two ways** of finding the missing addend in the addition table as seen below:

+	0	1	2	3	$\boxed{4}$
0	0	1	2	3	4
1	1	2	3	4	5
2	2	3	4	5	6
3	3	4	5	6	7
$\boxed{4}$	4	5	6	7	8

Finding The Missing Addend: $3 + \square = 7$

* Find 3 **in the top** row and go down the column, **stop at 7.** Then go across the row **to the left. The missing addend is the number at the end of the row.**
* Or find 3 **at the left** side column and go across the row, **stop at 7.** Then **go straight up** to the top. **The missing addend is the number at the top.**

The missing addend is the difference.

100 Basic Subtraction Facts

0 - 0 = 0	1 - 1 = 0	2 - 2 = 0	3 - 3 = 0	4 - 4 = 0
1 - 0 = 1	2 - 1 = 1	3 - 2 = 1	4 - 3 = 1	5 - 4 = 1
2 - 0 = 2	3 - 1 = 2	4 - 2 = 2	5 - 3 = 2	6 - 4 = 2
3 - 0 = 3	4 - 1 = 3	5 - 2 = 3	6 - 3 = 3	7 - 4 = 3
4 - 0 = 4	5 - 1 = 4	6 - 2 = 4	7 - 3 = 4	8 - 4 = 4
5 - 0 = 5	6 - 1 = 5	7 - 2 = 5	8 - 3 = 5	9 - 4 = 5
6 - 0 = 6	7 - 1 = 6	8 - 2 = 6	9 - 3 = 6	10 - 4 = 6
7 - 0 = 7	8 - 1 = 7	9 - 2 = 7	10 - 3 = 7	11 - 4 = 7
8 - 0 = 8	9 - 1 = 8	10 - 2 = 8	11 - 3 = 8	12 - 4 = 8
9 - 0 = 9	10 - 1 = 9	11 - 2 = 9	12 - 3 = 9	13 - 4 = 9

5 - 5 = 0	6 - 6 = 0	7 - 7 = 0	8 - 8 = 0	9 - 9 = 0
6 - 5 = 1	7 - 6 = 1	8 - 7 = 1	9 - 8 = 1	10 - 9 = 1
7 - 5 = 2	8 - 6 = 2	9 - 7 = 2	10 - 8 = 2	11 - 9 = 2
8 - 5 = 3	9 - 6 = 3	10 - 7 = 3	11 - 8 = 3	12 - 9 = 3
9 - 5 = 4	10 - 6 = 4	11 - 7 = 4	12 - 8 = 4	13 - 9 = 4
10 - 5 = 5	11 - 6 = 5	12 - 7 = 5	13 - 8 = 5	14 - 9 = 5
11 - 5 = 6	12 - 6 = 6	13 - 7 = 6	14 - 8 = 6	15 - 9 = 6
12 - 5 = 7	13 - 6 = 7	14 - 7 = 7	15 - 8 = 7	16 - 9 = 7
13 - 5 = 8	14 - 6 = 8	15 - 7 = 8	16 - 8 = 8	17 - 9 = 8
14 - 5 = 9	15 - 6 = 9	16 - 7 = 9	17 - 8 = 9	18 - 9 = 9

Subtraction Properties & Subtraction Facts (Review "Properties" p.120)

Like the addition facts, **the subtraction facts must also be memorized!** You can use **two subtraction properties** to reduce the number of subtraction facts to be mastered.

$0 - 0 = 0$ $1 - 0 = 1$ $2 - 0 = 2$ $3 - 0 = 3$	The **identity property of subtraction** says, **"A number minus zero is the number."** Knowing this property helps you to omit **10 facts** from the list. They are the facts found inside the box **(p.152)**
$\boldsymbol{n - 0 = n}$	**Generalization:** ***n*** can be ***any*** number - 99 or 278.
$1 - 1 = 0$ $2 - 2 = 0$ $3 - 3 = 0$ $4 - 4 = 0$	The **zero property of subtraction** says, **"A number minus itself equals 0."** If you know this property, you can omit **another 9 facts** - the facts inside the box on page 152.
$\boldsymbol{m - m = 0}$	**Generalization:** ***m*** can be ***any*** number - 35 or 999.

Remember: Mathematical properties generally have wide applications. Knowing them now will help you in your study of math in the days to come.

Rearranging the Subtraction Facts

[2 - 1 = 1]	* 7 - 1 = 6	* 9 - 3 = 6	* 11 - 4 = 7	* 14 - 5 = 9
	7 - 6 = 1	9 - 6 = 3	11 - 7 = 4	14 - 9 = 5
* 3 - 1 = 2	* 7 - 2 = 5	* 9 - 4 = 5	* 11 - 5 = 6	* 14 - 6 = 8
3 - 2 = 1	7 - 5 = 2	9 - 5 = 4	11 - 6 = 5	14 - 8 = 6
	* 7 - 3 = 4			[14 - 7 = 7]
* 4 - 1 = 3	7 - 4 = 3	* 10 - 1 = 9	* 12 - 3 = 9	
4 - 3 = 1		10 - 9 = 1	12 - 9 = 3	* 15 - 6 = 9
[4 - 2 = 2]	* 8 - 1 = 7	* 10 - 2 = 8	* 12 - 4 = 8	15 - 9 = 6
	8 - 7 = 1	10 - 8 = 2	12 - 8 = 4	* 15 - 7 = 8
* 5 - 1 = 4	* 8 - 2 = 6	* 10 - 3 = 7	* 12 - 5 = 7	15 - 8 = 7
5 - 4 = 1	8 - 6 = 2	10 - 7 = 3	12 - 7 = 5	
* 5 - 2 = 3	* 8 - 3 = 5	* 10 - 4 = 6	[12 - 6 = 6]	* 16 - 7 = 9
5 - 3 = 2	8 - 5 = 3	10 - 6 = 4		16 - 9 = 7
	[8 - 4 = 4]	[10 - 5 = 5]	* 13 - 4 = 9	[16 - 8 = 8]
* 6 - 1 = 5			13 - 9 = 4	
6 - 5 = 1	* 9 - 1 = 8	* 11 - 2 = 9	* 13 - 5 = 8	* 17 - 8 = 9
* 6 - 2 = 4	9 - 8 = 1	11 - 9 = 2	13 - 8 = 5	17 - 9 = 8
6 - 4 = 2	* 9 - 2 = 7	* 11 - 3 = 8	* 13 - 6 = 7	
[6 - 3 = 3]	9 - 7 = 2	11 - 8 = 3	13 - 7 = 6	[18 - 9 = 9]

You may find it easier to learn each pair of subtraction facts together (*). **Each pair is related to one addition facts.** (See p.150)

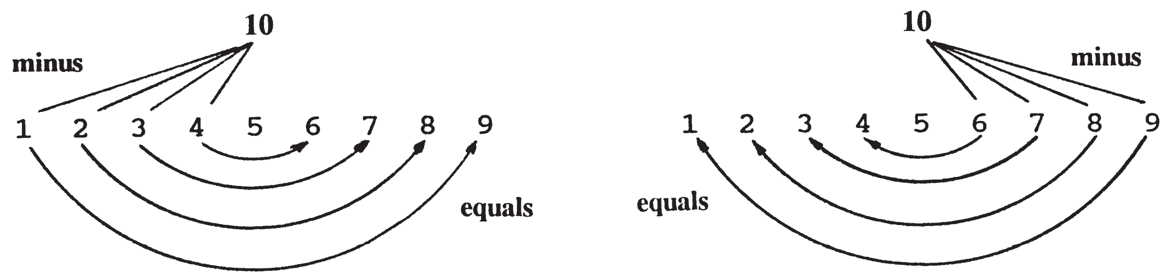
Subtraction Model 1 - Subtracting from 10 (Review first "Place Value" p.18)

Many students have difficulties learning the subtraction facts found inside the boundary (previous page) because **the subtrahend is larger than the minuend in the ones place.** If you are one of them, try one of the following methods.

Method 1. Subtracting From 10:

13 − 5	**Think:** 13 = 10 + 3
(10 - 5) + 3	Subtract 5 from 10 instead of 13.
5 + 3 = 8	Then add the difference to 3.

Learn to subtract from 10: (See also "Add Up To 10" p 134)



Subtraction Model 2 - Adding to 10 (See also "Playing With Numbers" pp.136-137)

$$\begin{array}{r} 3 \\ -\ 2 \\ \hline 1 \end{array} \quad \begin{array}{c} (3 + 2) \\ (2 + 2) \\ \\ \end{array} \quad \begin{array}{r} 5 \\ -\ 4 \\ \hline 1 \end{array}$$

$$1 = 1$$

The example shows, **adding the same number to *both*** minuend and subtrahend **does not change** the difference.

Model 2. Adding to 10:

17 − 9	**Since subtracting 10 is easy, change 9 to 10.**
(17 + 1) − (9 + 1)	**Add 1 to *both* minuend and subtrahend.**
18 − 10 = 8	**Then subtract 10 from 18.**

Note: This method is useful if the subtrahend **is close to 10,** such as 8 or 9.

Remember: Each subtraction model is trying to manipulate numbers to make it easier to subtract. So, **make sure you learn it right and do it correctly!**

Subtraction Model 3 - Subtracting Twice

Model 3. Subtracting Twice:

15 − 8	**Think: 5 – 8. 5 is not enough. 5 needs 3 more to make it 8.**
$\boxed{\begin{array}{l}(8 - 5 = 3)\\(5 + 3 = 8)\end{array}}$	**Actually, you are finding the difference between 8 and 5. Or You remember 5 + 3 = 8.**
10 − 3 = 7	**Then you take the 3 needed from 10.**

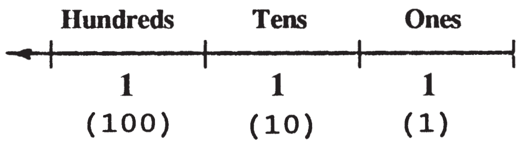
Comparing Model 1 and 3:
Both models deal with subtraction in which the subtrahend is larger than the minuend. But here is **the difference between the two:**

* **In Model 1: You start at tens place.** You subtract from 10, then add the difference to the number in the ones place.

* **In Model 3: You start at the ones place.** You take from the number in the ones place, then take what is short of from 10.

Borrowing In Subtraction & Decimal System (Review "Carrying" p.140)

To understand the concept of borrowing, you must first understand our number system. According to **base-10 system,** if the sum of any place is 10 or more, you ***carry*** the 10 to the next higher place by adding 1; if carry 20, add 2, etc. ***Borrowing*** **is just the opposite of carrying** as seen below:



Remember that number in any place is **10 times larger** than the number ***to its right.***

If you ***borrow*** **1 from the tens place,** you get **10 ones.**
(In *carrying*, 10 ones becomes 1 ten.)

If you ***borrow*** **1 from the hundreds place,** you get **10 tens** or **9 tens and 10 ones** (9 tens and 10 ones = 10 tens = 100)
(In *carrying,* 10 tens becomes 1 hundred.)

Remember, *in borrowing,* we do not take away anything from the number. **We just break the number apart, and regroup the number to make the subtraction possible.**

Borrowing In Subtraction - Using Expanded Form (Review "Expanded Form" p.24)

Students, in general, find borrowing harder to do than carrying. In carrying, you just add the numbers. But in borrowing, you have to **regroup numbers** which may involve more than one place.

(a)
$$\begin{array}{r} \downarrow \\ 34 \\ -\ \ 9 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} \downarrow\ \ \\ 319 \\ -\ \ 64 \\ \hline \end{array}$$

Borrowing becomes necessary **when you try to subtract a larger number from a smaller number.** For example:
- In (a), it happens in the *ones place.*
- In (b), it happens in the *tens place.*

If we write the numbers in expanded form, it would help us to see the process of borrowing. Let's use the example (a) above.

Standard Method **Expanded Method**

$$30 = 20 + 10$$

$$\begin{array}{r} {\scriptstyle 2\ \ 14} \\ \cancel{3}\ 4 \\ -\ \ \ 9 \\ \hline 2\ 5 \end{array} = \begin{array}{r} 30 + 4 \\ -\qquad 9 \\ \hline \end{array} = \begin{array}{r} 20 + 14 \\ -\qquad 9 \\ \hline 20 + \ \ 5 \end{array} = 25$$

Since 4 - 9 is impossible, we have to ***borrow*** **1 from the tens place.** That means, we **borrow 10 from 30 to make 14 in the ones place.**

Borrowing Across Zeros - Using Expanded Form

Example: Subtract. 502 - 7

a) 502 - 7

500 + 00 + 2

b) 400 + 100 + 2

c) 400 + 90 + (10 + 2)

502 - 7 = 495

Since 2 - 7 is impossible, we have to ***borrow*** **one 10 from the tens place.** Let's use expanded form to see how it works.

Since the tens-place is ***empty,*** we have to ***borrow*** **a 100 from the hundreds place** to fill the tens place.

Next, we ***borrow*** **a 10 from the tens place** and **add to the 2 in the ones place** to make it 12. Now, we can subtract 7 from 12.

Shortcut:

* To subtract across zero like 502 - 7, think of ***borrowing*** ***1 from 50,*** and change 50 to 49 with 12 in the ones place:

4 9 12
5 0 2

* To subtract across zeros like 3005 - 8, think of ***borrowing*** ***1 from 300,*** and change 300 to 299 with 15 in the ones place:

2 9 9 15
3 0 0 5

General Rules For Subtracting Numbers

The larger number is always at the top.
The digits must be lined up correctly.

At any place, if the minuend is smaller than subtrahend, borrow 1 from the number in the next higher place. Then reduce that number by 1.

$$\begin{array}{rr} 3 & {}^{1}7 \\ 4 & 7 \\ -\ 1 & 9 \\ \hline 2 & 8 \end{array}$$

Begin at the right – subtract the ones, then the tens,.... until every place has been subtracted.

To check: **Add the difference to the subtrahend. The sum should equal the minuend.**

Remember: No matter how large the number may be, you subtract **only two numbers at a time** (See p.123). And each time you use the subtraction facts to compute. The fact is, you **can do any** subtraction problem with confidence, **if you have mastered two things:**

1. **Basic subtraction facts** (See p.154)
2. **The skill of borrowing** (See p.158)

Subtraction - Borrowing Twice (Review first p.158)

The **speed** and **accuracy** with which you subtract depend on your knowledge of the basic subtraction facts and the skill of borrowing. Remember, accuracy always comes first! Example: Subtract. 625 - 147.

$$\begin{array}{r} {\scriptstyle 1\,15} \\ 6\cancel{2}5 \\ -\ 147 \\ \hline 8 \end{array}$$

Step 1. Subtract the ones: 5 - 7 = impossible.

* Borrow 1 from the tens place: **2 became 1.**
  Strike out 2, write 1 above it.
* **15** - 7 = 8. Write 8 in the ones place under 7.

$$\begin{array}{r} {\scriptstyle 5\ 11} \\ \cancel{6}25 \\ -\ 147 \\ \hline 78 \end{array}$$

Step 2. Subtract the tens: 1 - 4 = impossible.

* Borrow 1 from the hundreds place: **6 became 5.**
  Strike out 6, write 5 above it.
* **11** - 4 = 7. Write 7 in the tens place under 4.

$$\begin{array}{r} {\scriptstyle 5\quad} \\ \cancel{6}25 \\ -\ 147 \\ \hline 478 \end{array}$$

Step 3. Subtract the hundreds.

* **5** - 1 = 4. Write 4 in the hundreds place under 1.

Check: The difference plus subtrahend should equal minuend.

(difference) + (subtrahend) = (minuend)

478 + 147 = 625 ✓

Subtracting Across Zeros (See also p.160)

Subtract. 500 - 316. Remember, **"0s" are place holders** which means the ones and the tens places are **open** for regrouping.

$$\begin{array}{r} {\scriptstyle 4}\;\;{\scriptstyle 9}\;\;{\scriptstyle 10} \\ \cancel{5}\;\;\cancel{0}\;\;0 \\ -\;3\;\;1\;\;6 \\ \hline \end{array}$$

Step 1. Regrouping the tens and ones places.

* First, borrow 1 from the hundreds place, **5 becomes 4,** and the **1 borrowed becomes 10** in the tens-place.
* Again, borrow 1 from the tens place, **10 becomes 9,** and the **1 borrowed becomes 10** in the ones place.

$$\begin{array}{r} {\scriptstyle 4}\;\;{\scriptstyle 9}\;\;{\scriptstyle 10} \\ \cancel{5}\;\;\cancel{0}\;\;0 \\ -\;3\;\;1\;\;6 \\ \hline 1\;\;8\;\;4 \end{array}$$

Step 2. Subtract.

* Subtract ones place: 10 - 6 = 4. Write 4 in the ones place under 6.
* Subtract tens place: 9 - 1 = 8 Write 8 in the tens place under 1.
* Subtract hundreds place: 4 - 3 = 1. Write 1 in the hundreds place under 3.

Check: 184 + 316 = 500. ✓

Shortcut: To borrow across zeros like 500, think of borrowing 1 from 50 and **change 50 to 49** with **10 in the ones place.**

Summary
(Subtraction)

* Subtraction and addition are inverse operations. There are two subtraction facts for every addition fact.
* We can use the addition table to find the difference. To find the difference is the same as to find the missing addend.
* We can always add but we can't always subtract because subtraction is not commutative. If we subtract a larger number from a smaller number, the answer will be a "negative" number
* Subtraction facts are important. We use these facts to subtract large and small numbers; only two numbers at a time. These facts are also used in division.
* Borrowing is the opposite of carrying. In borrowing, we regroup the number to make the subtraction possible.
* In subtracting numbers, line up the numbers correctly in vertical form with the larger number at the top. Then, subtract the ones, followed by the tens, always in that order At any place when the minuend is smaller than subtrahend, borrow 1 from the number in the next larger place and reduce that number by 1
* To check, add the difference to the subtrahend. The sum should equal the minuend.

Part II. Whole Number Operations

D. Multiplication

Table of Contents

Multiplication - A Repeated Addition (see p.110)

Suppose you bought a box of chocolate which had 6 rows with 8 pieces in each row. If you want to know **how many pieces of chocolate are altogether,** you can use one of the following methods to find out:

By Adding:

$$\underbrace{8 + 8 + 8 + 8 + 8 + 8}_{\textbf{6 addends}\text{ (6 rows)}} = \underbrace{48}_{\textbf{sum}\text{ (total)}}$$

By Multiplying:

$$\underbrace{8}_{\substack{\textbf{multiplicand}\\ \text{(8 pieces)}}} \times \underbrace{6}_{\substack{\textbf{multiplier}\\ \text{(6 rows)}}} = \underbrace{48}_{\substack{\textbf{product}\\ \text{(total)}}}$$

Read: "8 times 6 equals 48" or **"6 eights equal 48."**

It's **much faster to use multiplication** when we have to **add the same number many times.**

Factors & Multiples (Read also pp.282-283)

Factors are numbers that you multiply together to get products. Therefore, factors are another name for **multiplicand** and **multiplier. Multiple** is another name for product but with this difference:

factor (multiplicand)		**factor** (multiplier)		**product**
3	x	1	=	3
3	x	2	=	6
3	x	3	=	9
3	x	4	=	12
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

multiples of 3

The difference between product and multiple:

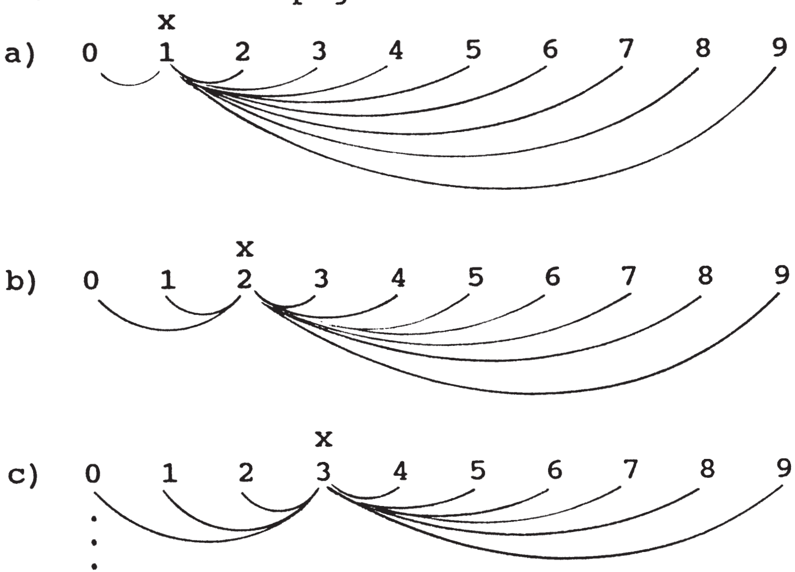
* **3, 6, 9, 12, ..., *each* is a product of two factors.**
Factors come in pairs (two) because we multiply only two numbers at a time.

* **3, 6, 9, 12, ..., are *all* multiples of 3.**

Connection: Factors and multiples are important concepts in fractions.

Multiplication Facts: Combinations of 10 Digits (See also p.128)

The multiplication facts, like the addition facts, are the combinations of the ten digits (0, 1, 2,..., 9). **Skip 0** because 0 times any number is 0. **Start with 1. Multiply 1 to each digit** ***including itself*** like a). Follow the same procedure for each digit, and you will have the 90 multiplication facts listed on next page.



1 x 0
1 x 1
1 x 2
1 x 3

Do you see some facts repeat?

1 x 2 = 2 x 1

2 x 0
2 x 1
2 x 2

1 x 3 = 3 x 1

Commutative prop. (See p.172)

3 x 0
3 x 1
3 x 2

90 Basic Multiplication Facts -- One-Digit Multiplication

Any number times zero is zero!

1 x 0 = 0	2 x 0 = 0	3 x 0 = 0	4 x 0 = 0
1 x 1 = 1	2 x 1 = 2	3 x 1 = 3	4 x 1 = 4
1 x 2 = 2	2 x 2 = 4	3 x 2 = 6	4 x 2 = 8
1 x 3 = 3	2 x 3 = 6	3 x 3 = 9	4 x 3 = 12
1 x 4 = 4	2 x 4 = 8	3 x 4 = 12	4 x 4 = 16
1 x 5 = 5	2 x 5 = 10	3 x 5 = 15	4 x 5 = 20
1 x 6 = 6	2 x 6 = 12	3 x 6 = 18	4 x 6 = 24
1 x 7 = 7	2 x 7 = 14	3 x 7 = 21	4 x 7 = 28
1 x 8 = 8	2 x 8 = 16	3 x 8 = 24	4 x 8 = 32
1 x 9 = 9	2 x 9 = 18	3 x 9 = 27	4 x 9 = 36

5 x 0 = 0	6 x 0 = 0	7 x 0 = 0	8 x 0 = 0	9 x 0 = 0
5 x 1 = 5	6 x 1 = 6	7 x 1 = 7	8 x 1 = 8	9 x 1 = 9
5 x 2 = 10	6 x 2 = 12	7 x 2 = 14	8 x 2 = 16	9 x 2 = 18
5 x 3 = 15	6 x 3 = 18	7 x 3 = 21	8 x 3 = 24	9 x 3 = 27
5 x 4 = 20	6 x 4 = 24	7 x 4 = 28	8 x 4 = 32	9 x 4 = 36
5 x 5 = 25	6 x 5 = 30	7 x 5 = 35	8 x 5 = 40	9 x 5 = 45
5 x 6 = 30	6 x 6 = 36	7 x 6 = 42	8 x 6 = 48	9 x 6 = 54
5 x 7 = 35	6 x 7 = 42	7 x 7 = 49	8 x 7 = 56	9 x 7 = 63
5 x 8 = 40	6 x 8 = 48	7 x 8 = 56	8 x 8 = 64	9 x 8 = 72
5 x 9 = 45	6 x 9 = 54	7 x 9 = 63	8 x 9 = 72	9 x 9 = 81

Multiplication Properties & Multiplication Facts (Review "Properties" p.121)

Learn the mathematic properties, and use them to your advantage! For example, if you know the multiplication properties, you can cut the need for memorization in half:

1 x 0 = 0 2 x 0 = 0 3 x 0 = 0	The **zero property of multiplication** says, **"A number times 0 is 0."** If you know the property, you don't have to memorize 9 of the facts.
***n* x 0 = 0**	**Generalization:** ***n*** can be any number -- 33, 128, etc.
1 x 1 = 1 1 x 2 = 2 1 x 3 = 3	The **identity property of multiplication** says, **"Any number multiplied by 1 is the number."** Knowing this property, you can omit another 9 facts.
1 x *n* = *n*	**Generalization:** ***n*** can be any number -- 99, 9,999, etc.
2 x 3 = 3 x 2 2 x 4 = 4 x 2 3 x 6 = 6 x 3	The **commutative property of multiplication** says, **"The order in** which you multiply the numbers **does not affect the answer."** Therefore, you have only 36 **facts** to remember.
a x *b* = *b* x *a*	**Generalization:** ***a*** and ***b*** can be any number, **except 0.**

Rearranging The Multiplication Facts

Attention Please! **Your ability to do division and fractions depend on your knowledge of multiplication facts.** Memorize them now so that you will enjoy working with division and fractions later. If you know the multiplication properties, all you need to memorize are the following facts:

* 2 x 2 = 4	* 3 x 3 = 9	* 4 x 4 = 16
2 x 3 = 3 x 2 = 6	3 x 4 = 4 x 3 = 12	4 x 5 = 5 x 4 = 20
2 x 4 = 4 x 2 = 8	3 x 5 = 5 x 3 = 15	4 x 6 = 6 x 4 = 24
2 x 5 = 5 x 2 = 10	3 x 6 = 6 x 3 = 18	4 x 7 = 7 x 4 = 28
2 x 6 = 6 x 2 = 12	3 x 7 = 7 x 3 = 21	4 x 8 = 8 x 4 = 32
2 x 7 = 7 x 2 = 14	3 x 8 = 8 x 3 = 24	4 x 9 = 9 x 4 = 36
2 x 8 = 8 x 2 = 16	3 x 9 = 9 x 3 = 27	
2 x 9 = 9 x 2 = 18		7 x 8 = 8 x 7 = 56
	* 6 x 6 = 36	7 x 9 = 9 x 7 = 63
* 5 x 5 = 25	6 x 7 = 7 x 6 = 42	
5 x 6 = 6 x 5 = 30	6 x 8 = 8 x 6 = 48	* 8 x 8 = 64
5 x 7 = 7 x 5 = 35	6 x 9 = 9 x 6 = 54	8 x 9 = 9 x 8 = 72
5 x 8 = 8 x 5 = 40		
5 x 9 = 9 x 5 = 45	* 7 x 7 = 49	* 9 x 9 = 81

Facts with "*" are squared numbers (See p.41)

Suggestion: In practicing, say facts **both ways** before giving the answer. It helps you to remember the commutative property.

Multiplication Table

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

– factors

factors

Study the table carefully. Do you see the multiplication properties? Do you see different pairs of factors that give the same product?

Multiples: A Pattern In Multiplication Table (See also "Multiples" p.283)

In the multiplication table, **the numbers in each row are the multiples of the number heading the row;** and **the numbers in each column are the multiples of the number heading the column.** Let's take the row of 5 as an example:

x	0	1	2	3	4...
	(5 x 0)	(5 x 1)	(5 x 2)	(5 x 3)	(5 x 4)...
5	0	5	10	15	20...
		+5	+5	+5	+5...

Multiply 5 by 1, by 2,...

All multiples of 5

Count by 5's → + 5.

Two Ways of Finding the Multiples of A Number:

1. **Multiply the number by 1, by 2,..., or by any whole number.** **All** the products are the multiples of the number. **Or**
2. **Count by the number (skip count),** say **count by 5's:** 5, 10, 15, 20,...**(repeatedly add 5)**. They are multiples of 5.

Using The Multiplication Table To Find Products (Compare with p.133)

In the multiplication table, **the digits at the top** and **the digits at the left side are factors.** The rest of the numbers are the products of two factors. The procedure of finding the product of two numbers is the same as finding the sum of two addends. There are **two ways:**

x	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	$\boxed{12}$	15
4	4	8	$\boxed{12}$	16	20

Finding The Product: $4 \times 3 = \square$

* Find the factor 4 **in the top row,** and the factor 3 **in the left column.** Then go down the column of 4 and go across the row of 3. **The product is the number where the column and the row meet. Or**
* Find 4 in the left column and 3 in the top row. The product is the number where the row and the column meet.

Observe The Pattern: (see p.174)

$$4 \times 3 = \begin{cases} (4 \times 2) + \square = 12 \\ (4 \times 4) - \square = 12 \end{cases}$$

Can you find the missing numbers? Do you see the pattern?

Playing With Numbers (II) (Read also p.136)

Since any number **larger than 1** can be written as **a product of two factors** or **a sum of two smaller numbers,** we can play with either factor. For example:

a) $4 \times 6 = 4 \times \underline{3 \times 2} = 24$

Think: 4 x 6 is the **double** of 4 x 3, because **6 = 3 x 2.**

$4 \times 6 = 4 \times 3 \times \boxed{} = 24$

How about removing 2 and ask your friend to find the missing number.

b) $4 \times 6 = 4 \times (5 + 1) = 24$
$= 4 \times 5 + 4 \times 1 = 24$

Write **6 as (5 + 1), using a parenthesis.** Then use **distributive property.** (See p.121)

$4 \times 6 = 4 \times (5 + \boxed{}) = 24$

How about removing 1 and ask your friend to find the missing number.

c) $8 \times 9 = (8 \times 10) - 8 = 72$

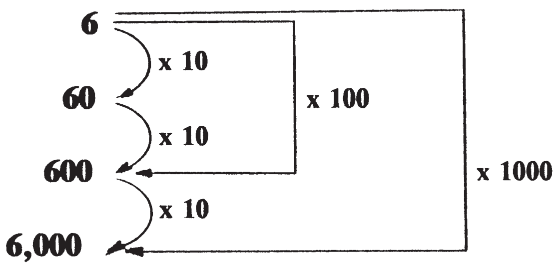
$= 8 \times (10 - 1)$

$= \underline{8 \times 10} - \underline{8 \times 1} = 72$

Since **8 x 9 + 8** = 8 x 10, we can write 8 x 9 as **8 x 10 - 8.**
Or write **9 as (10 - 1)** and then use **distributive property.**
Either way gives the same answer.

Multiplying by 10, 100, 1000, ... (See also p.67)

Since our number system is based on 10, multiplying a number by 10, 100, 1000, ... is very easy to work with. ***Memorize the rules given below!*** The rule is very useful in multiplication/division of **whole numbers** and **decimals;** also in **scientific notation.**



Remember: Any number ending with zero(s) **has a factor of 10, 100, 1000,...**

60 = 6 x 10
600 = 6 x 100
6,000 = 6 x 1000

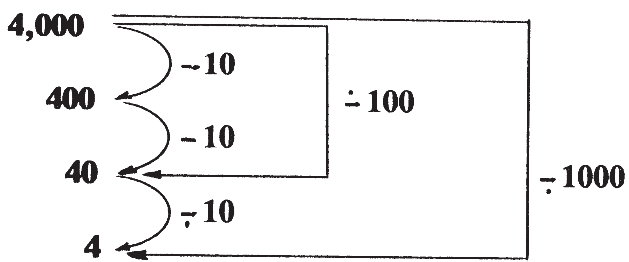
Rule:

To multiply by 10, 100, 1000, etc., add the number of zeros in the multiplier to the multiplicand:

* multiply by 10, add 1 zero. *(Add 1 zero = multiply by 10.)*
* multiply by 100, add 2 zeros. *(Add 2 zeros = multiply by 100.)*
* multiply by 1000, add 3 zeros. *(Add 3 zeros = multiply by 1000.)*

Dividing By 10, 100, 1000, ... (Compare with "Multiplying By 10, 100,...)

This page is put here for the purpose of comparing with the opposite page. Dividing a number which ends in zeros by 10, 100, etc. is the opposite of multiplying a number by 10, 100, ... Either operation can be done mentally.



Writing the division in fraction also help us to understand:

$$\frac{4000}{10} = \frac{400 \times \cancel{10}}{\cancel{10}} = 400$$

$$\frac{40\cancel{00}}{1\cancel{00}} = 40$$

Rule:

To divide a number which ends in zeros by 10, 100, etc., cancel the same number of zeros in both the divisor and the dividend:

* Divide by 10, cancel 1 zero. *(Drop 1 zero = divide by 10)*
* divide by 100, cancel 2 zeros. *(Drop 2 zeros = divide by 100)*

Multiplying By Multiples of 10, 100,... (See also "Multiples" p.283)

Multiples of 10 are: 10, 20, 30, 40,... (Multiply 10 by 1, by 2,...)
Multiples of 100 are: 100, 200, 300,... (multiply 100 by 1, by 2,...)

a) $25 \times 30 = 25 \times \underline{3 \times 10} = 750$ **Multiply 25 x 3, then add 1 zero.**

b) $43 \times 200 = 43 \times \underline{2 \times 100} = 8600$ **Multiply 43 x 2, then add 2 zeros.**

Multiplying Numbers Ending With Zeros

c)
$$\begin{array}{r l}
30 & - (3 \times 10) \\
\times\ 50 & - (5 \times 10) \\
\hline
1500 & - (15 \times 100)
\end{array}$$

1st. Multiply 3 x 5 = 15.
2nd. Add the **total number of zeros** found in **two factors (2 zeros)** to 15.

d)
$$\begin{array}{r l}
150 & - (15 \times 10) \\
\times\ 500 & - (5 \times 100) \\
\hline
75000 & - (75 \times 1000)
\end{array}$$

1st. Multiply 15 x 5 = 75.
2nd. Add the **total number** of zeros in **two factors (3 zeros)** to 75.

Prior Knowledge For Multiplying Numbers (Review first "Place-Value" p.109)

If you answer **"yes"** to the following questions without hesitation, then you can do any multiplication, large or small, with confidence.

1. **Can you give multiplication facts quickly from memory with no error?** (p.173)
2. **Do you understand the concept of place value?** (p.109)
3. **Have you mastered the skill of carrying?** (p.140)
4. **Have you mastered the skill of addition?** (p.143)

Understand The Vocabulary of Multiplication:

$$\begin{array}{r c l}
2\,1\,3 & & \textbf{multiplicand}\ \text{(factor)} \\
\times\ \ 2\,1 & & \textbf{multiplier}\ \text{(factor)} \\ \cline{1-1}
2\,1\,3 & \longleftarrow & \textit{partial product}\ (213 \times 1 = 213) \\
+\ 4\,2\,6\ \ & \longleftarrow & \textit{partial product}\ (213 \times 2 = 426) \\ \cline{1-1}
4\,4\,7\,3 & & \textbf{product}\ (213 + 426 = 4473)
\end{array}$$

The partial products occur when the multiplier is two digits or more. Each time when we multiply the multiplicand by a digit of the multiplier, we get a partial product. Therefore, if multiplied by:

* a **two-digits** multiplier - we have **two partial products.**
* a **three-digits** multiplier - we have **three partial products.**

General Process Of Multiplying Numbers

We **multiply the top number (multiplicand) by each digit of the bottom number (multiplier) starting from the ones digit** as you see below. Example: Multiply 738 x 26

Step 1.

$$\begin{array}{r} 7\ 3\ 8 \\ \times\quad\quad 6 \\ \hline \end{array}$$

Step 1. 738 x 6 (1st digit of multiplier)
Begin with the *ones* digit of the multiplicand:
1st. 6 x 8 = 48
2nd. 6 x 3 = 18 (actually 6 x 30 = 180)
3rd. 6 x 7 = 42 (actually 6 x 700 = 42,000)
Remember always to add the carried over numbers.

Step 2.

$$\begin{array}{r} 7\ 3\ 8 \\ \times\quad 2\ 6 \\ \hline \end{array}$$

Step 2. 738 x 2 (2nd digit of multiplier)
Again, begin with the *ones* digit of the multiplicand:
1st. 2 x 8 = 16 (actually 20 x 8 = 160)
2nd. 2 x 3 = 6 (actually 20 x 30 = 600)
3rd. 2 x 7 = 14 (actually 20 x 700 = 14,000)

Note: We multiply **larger numbers *in exactly the same way* we multiply smaller numbers. The larger numbers have more digits, that's all.** If it were a three-digit multiplier, you continue the process by multiplying the top number by the 3rd digit of the multiplier, and so on.

General Rule For Placing The Partial Products (Review the previous pages first)

Since our number system is a **place value system, it is important that each partial product be placed in a proper place.** Let's use 1357 x abc for an illustration:

```
        1 3 5 7
      x   c b a
      ---------
        a a a a    ①
      b b b b      ②
  + c c c c        ③
  -------------
    p p p p p p    ④
```

Study "Multiplying by 10, 100,..." p.178 and "Multiplying By Three-Digits" p.186.

Step 1. Multiply 1357 by a ***(ones).***
Write the 1st partial product under the line **with the first digit in the *ones*** place. ①

Step 2. Multiply 1357 by b ***(tens)***
Write the 2nd partial product under the 1st one **with the first digit *in the same column with b*** **in the multiplier** – ***tens*** place. ②

Step 3. Multiply 1357 by c ***(hundreds)***
Write the 3rd partial product under the 2nd one **with the first digit *in the same column with*** **c in the multiplier** – ***hundreds*** place. ③

The product of 1357 x abc is ***the sum*** **of all the partial products.** ④ ***Make sure all the digits are lined up correctly*** before adding.

Multiplying By One-Digit Numbers With Carrying (Review first pp.182-183)

```
    2 4
    7 3 8
x       6
---------
  4 4 2 8
```

1st. Multiply 6 x 8 *(ones)* = 48. (48 = 40 + 8).
Write 8 in the ones place under 6.
Carry 4 (40) to the tens place. Write 4 above 3.

2nd. Multiply 6 x 3 *(tens)* = 18. (18 means 100 + 80)
Add 18 + 4 carried = 22. (22 means 200 + 20)
Write 2 in the tens place at the left of 8.
Carry 2 (200) to the hundreds place. Write 2 above 7.

3rd. Multiply 6 x 7 *(hundreds)* = 42. (42 means 4000 + 200)
Add 42 + 2 carried = 44. (44 means 4000 + 400)
Since there is no digit in the thousands place, write 44 (means 4400) to the left of 2.

Do you notice that we used one-digit multiplication facts in each step? **The fact is you will be using these basic facts in doing all multiplication/division of whole numbers, decimals, and fractions.** Memorize the facts now, if you haven't done so!

Multiplication Without Carrying - In case of multiplication without carrying, write the product of each place (ones, tens,...) under that very same place. (ones, tens,...).

Multiplying By Two-Digit Numbers With Carrying (Review first pp.182-183)

```
      6 9 7    Same as     697        697
  x     3 2              x  30      x   2
    1 3 9 4              20910  +   1394   =  22304
+ 2 0 9 1 0
  2 2 3 0 4               Step 2     Step 1
```

Step 1. Multiply 697 by 2 *(ones)*. **(See the previous page.)**

Step 2. Multiply 697 by 3 *(tens)*. (Remember, 3 in the tens place = 30)

1st. Multiply 3 x 7 *(ones)* = 21. (actually 30 x 7 = 210)

Write 1 (means 10) in the ***tens*** place under 9. (Omit 0 in ones place.)

Carry 2 (means 200) **to the hundreds place.** (See next page.)

2nd. Multiply 3 x 9 *(tens)* = 27. (actually 30 x 90 = 2700)

Add 27 + 2 carried = 29. (actually 29 means 2000 + 900)

Write 9 in the hundreds place under 3, at the left of 1.

Carry 2 (means 2000) **to the thousands place.**

3rd. Multiply 3 x 6 *(hundreds)* = 18. (actually 30 x 600 = 18,000)

Add 18 + 2 carried = 20. (20 here means 20,000)

Write 20 at the left of 9.

Multiplying By Two-Digit Numbers With Carrying (Review first pp.182-183)

$$
\begin{array}{r}
697 \\
\times\ \ 32 \\
\hline
1394 \\
+\ 20910 \\
\hline
22304
\end{array}
\qquad \text{Same as} \qquad
\begin{array}{r}
697 \\
\times\ 30 \\
\hline
20910
\end{array}
\ + \
\begin{array}{r}
697 \\
\times\ 2 \\
\hline
1394
\end{array}
\ = \ 22304
$$

Step 2   Step 1

Step 1. Multiply 697 by 2 *(ones)*. (See the previous page.)

Step 2. Multiply 697 by 3 *(tens)*. (Remember, 3 in the tens place = 30)

1st. Multiply 3 x 7 *(ones)* = 21. (actually 30 x 7 = 210)

Write 1 (means 10) in the ***tens*** place under 9. (Omit 0 in ones place.)

Carry 2 (means 200) **to the hundreds place.** (See next page.)

2nd. Multiply 3 x 9 *(tens)* = 27. (actually 30 x 90 = 2700)

Add 27 + 2 carried = 29. (actually 29 means 2000 + 900)

Write 9 in the hundreds place under 3, at the left of 1.

Carry 2 (means 2000) **to the thousands place.**

3rd. Multiply 3 x 6 *(hundreds)* = 18. (actually 30 x 600 = 18,000)

Add 18 + 2 carried = 20. (20 here means 20,000)

Write 20 at the left of 9.

Multiplying By Three-Digit Numbers (Review first the last two pages.)

```
      5 4 2
    x 4 3 2
    -------
    1 0 8 4   = 542 x 2         First partial product   (Step 1)
  1 6 2 6     = 542 x 3 (30)    Second partial product  (Step 2)
+ 2 1 6 8     = 542 x 4 (400)   Third partial product   (Step 3)
-----------
  2 3 4 1 4 4                   The product
```

Step 1. Multiply 542 by 2 *(ones)*. (See "Multiplying By One-Digit")
542 x 2 = 1084. Write 1084, the first partial product, under the multiplier below the line with ***4 in the ones place under 2.***

Step 2. Multiply 542 by 3 *(tens)*. (See "Multiplying By Two-Digit")
542 x 3 = 1626. Write 1626 under the first partial product with ***6 in the same column (place) with 3 in the multiplier.***
Second partial product is actually 542 x 30 = 16260 with 0 in the ones place omitted because 4 + 0 = 4. (Adding zeros does not change the sum.)

Step 3. Multiply 542 by 4 *(hundreds)*. Follow same procedure.
542 x 4 = 2168. Write 2168 under the second partial product with ***8 in the same column (place) with 4 in multiplier***
Third partial product is actually 542 x 400 = 216800 with 0s in ones and tens places omitted.

Checking Multiplication (Review first "Checking Multiplication" p.115)

Making it a habit, always check your answers. You know by now, when comes to mathematics, the answer is ***either correct or incorrect.*** Mathematics is an exact science which knows no mercy! If you want to get good marks on tests or homework, **go over your work and check the following:**

* **Multiplication**	- Are multiplications done in order?
	- Are all one-digit multiplications correct?
* **Carrying**	- Are there numbers needed to be carried over?
* **Addition**	- Are numbers carried being added?
	- Are additions done correctly?
* **Place-Value**	- Are the product of each digit written in the correct value-place?
	- Are the partial products lined up correctly?

Suggestion: At the beginning of your study of multiplication, you should use one of the following methods to check your answer:

1. **(multiplicand) x (multiplier)** **–(change the place)→** **(multiplier) x (multiplicand)**

   Multiplication is commutative, the order of the factors does not affect the product.

2. **(Product) ÷ (multiplier) = (multiplicand)** or **(product) ÷ (multiplicand) = (multiplier)**

   Division and multiplication are inverse operations.

Summary
(Multiplication)

* Multiplication is a repeated addition. When adding the same number over more than once, multiplication is a faster way to compute.
* Multiplication facts are used not only in all multiplication problems, but also in divisions and fractions. Students should memorize the multiplication facts. Multiplication properties help us cut down the number of multiplication facts to be memorized.
* From the multiplication table, you can find the products of all one-digit multiplication facts, the multiplication properties, and different pairs of factors that give the same product.
* Powers of 10 is easy to work with. For example, to multiply a number by a power of 10, we just add to the multiplicand the exact number of zeros that are in the multiplier
* To multiply two-or more digit numbers, write the numbers vertically Then multiply the multiplicand, in turn, by each digit of the multiplier beginning at ones digit. Place each partial product in the proper place. The product is the sum of all partial products.
* To check multiplication, change the positions of the factors and multiply Multiplication is commutative.

Part II. Whole Number Operations

E. Division

Table of Contents

Division - A Repeated Subtraction (See also p.110)

If multiplication is a repeated addition, **division is a repeated subtraction.** Suppose you want to divide 48 pieces of candy among 12 students, you can use **either subtraction or division** to find out **how many pieces each student will get.**

Using Repeated Subtraction:

①	②	③	④
$48 - 12 = 36$	$36 - 12 = 24$	$24 - 12 = 12$	$12 - 12 = 0$

You keep taking 12 pieces away from 48, until there is nothing left. The number of times you subtract is the answer - 4 pieces. There are four 12 pieces in 48.

Using Division:

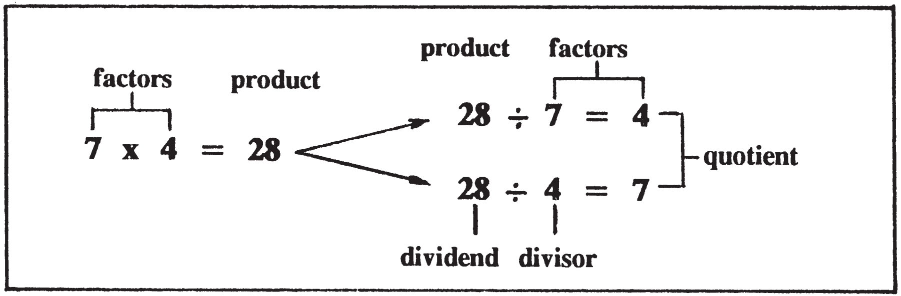
$48 - 12 = 4$ or $\frac{48}{12} = 4$

We are dividing 48, the total, into 12 equal parts, and we get 4 pieces.

Division is a much faster way to divide things/numbers into *equal parts*.

Division - An Inverse Operation of Multiplication (See also p.110)

Division is the **opposite** of multiplication. The following shows that for **every multiplication fact, except multiplication by zero, there are two related division facts:**



Although division and multiplication are inverse operations, **multiplication is commutative** ***while division is not.*** For example:

Multiplication:	$12 \times 2 = 24$	**and**	$2 \times 12 = 24$
Division:	$12 \div 2 = 6$	**but**	$2 \div 12 =$ impossible

Therefore, in division **with** ***whole numbers,*** **the dividend must be equal to or larger than the divisor.** (See p.122)

90 Basic Division Facts - One-Digit Divisor

Division by zero is impossible!

0 ÷ 1 = 0	0 ÷ 2 = 0	0 ÷ 3 = 0	0 ÷ 4 = 0
1 ÷ 1 = 1	2 ÷ 2 = 1	3 ÷ 3 = 1	4 ÷ 4 = 1
2 ÷ 1 = 2	4 ÷ 2 = 2	6 ÷ 3 = 2	8 ÷ 4 = 2
3 ÷ 1 = 3	6 ÷ 2 = 3	9 ÷ 3 = 3	12 ÷ 4 = 3
4 ÷ 1 = 4	8 ÷ 2 = 4	12 ÷ 3 = 4	16 ÷ 4 = 4
5 ÷ 1 = 5	10 ÷ 2 = 5	15 ÷ 3 = 5	20 ÷ 4 = 5
6 ÷ 1 = 6	12 ÷ 2 = 6	18 ÷ 3 = 6	24 ÷ 4 = 6
7 ÷ 1 = 7	14 ÷ 2 = 7	21 ÷ 3 = 7	28 ÷ 4 = 7
8 ÷ 1 = 8	16 ÷ 2 = 8	24 ÷ 3 = 8	32 ÷ 4 = 8
9 ÷ 1 = 9	18 ÷ 2 = 9	27 ÷ 3 = 9	36 ÷ 4 = 9

0 ÷ 5 = 0	0 ÷ 6 = 0	0 ÷ 7 = 0	0 ÷ 8 = 0	0 ÷ 9 = 0
5 ÷ 5 = 1	6 ÷ 6 = 1	7 ÷ 7 = 1	8 ÷ 8 = 1	9 ÷ 9 = 1
10 ÷ 5 = 2	12 ÷ 6 = 2	14 ÷ 7 = 2	16 ÷ 8 = 2	18 ÷ 9 = 2
15 ÷ 5 = 3	18 ÷ 6 = 3	21 ÷ 7 = 3	24 ÷ 8 = 3	27 ÷ 9 = 3
20 ÷ 5 = 4	24 ÷ 6 = 4	28 ÷ 7 = 4	32 ÷ 8 = 4	36 ÷ 9 = 4
25 ÷ 5 = 5	30 ÷ 6 = 5	35 ÷ 7 = 5	40 ÷ 8 = 5	45 ÷ 9 = 5
30 ÷ 5 = 6	36 ÷ 6 = 6	42 ÷ 7 = 6	48 ÷ 8 = 6	54 ÷ 9 = 6
35 ÷ 5 = 7	42 ÷ 6 = 7	49 ÷ 7 = 7	56 ÷ 8 = 7	63 ÷ 9 = 7
40 ÷ 5 = 8	48 ÷ 6 = 8	56 ÷ 7 = 8	64 ÷ 8 = 8	72 ÷ 9 = 8
45 ÷ 5 = 9	54 ÷ 6 = 9	63 ÷ 7 = 9	72 ÷ 8 = 9	81 ÷ 9 = 9

Division By Zero - Impossible! (See "Division Facts" p.194)

In studying mathematics, whenever you see words like "shortcut," "impossible," etc., **try, as much as you can, to understand the reason behind it.**

We have only 90 basic division facts instead of 100 because "division by 0 is impossible." Here is the reason why. Read carefully the following because it is **a logical reasoning.** Let's use $9 \div 0$ as an example:

Suppose we say	$9 \div 0 = n$	We use *n* to stand for the quotient.
Then	$n \times 0 = 9$	Because division and multiplication are **inverse operations** (See p.198).
The fact	$n \times 0 = 0$	We know **any number times zero is zero** ***not 9.*** (Zero property of multiplication)
Since	$n \times 0 \neq 9$	Which means **n x 0** ***can never*** **equal to 9.**
Therefore	$9 - 0$	*is impossible!*

Connection: We can not have ZERO as the denominator of a fraction, because division by zero is impossible.

Division Properties & Multiplication Properties (Review also p 121)

If you remember the following division properties, you do not have to memorize the facts found inside the boxes on page 194. Since division is an inverse operation of multiplication, their properties are related.

a) $0 \div n = 0$ * Zero property of division: **"Zero divided by any non-zero number is zero."** (*n* can be 1, 2, 3,. .)

$0 \times n = 0$ * Zero property of multiplication: **"Any number times zero is zero."** (*n* can be 0, 1, 2, 3,...)

b) $n \div 1 = n$ * Identity property of division: **"Any non-zero number divided by 1 is the number."** (*n* can be 1, 2,...)

$n \times 1 = n$ * Identity property of multiplication: **"Any number times 1 is the number."** (*n* can be 0, 1, 2,...)

c) $n \div n = 1$ * One property of division: **"Any non-zero number divided by itself is 1."** (*n* can be 1, 2, 3,...)

$1 \times n = n$ * Identity property of multiplication: **"Any number times 1 is the number."** (*n* can be 0, 1, 2,...)

Note that both division properties, b) and c), came from the same multiplication property (p.172).

Rearranging Division Facts

```
  [4 ÷ 2 = 2]
*  6 ÷ 2 = 3
   6 ÷ 3 = 2
*  8 ÷ 2 = 4
   8 ÷ 4 = 2
* 10 ÷ 2 = 5
  10 ÷ 5 = 2
* 12 ÷ 2 = 6
  12 ÷ 6 = 2
* 14 ÷ 2 = 7
  14 ÷ 7 = 2
* 16 ÷ 2 = 8
  16 ÷ 8 = 2
* 18 ÷ 2 = 9
  18 ÷ 9 = 2

 [36 ÷ 6 = 6]
* 42 ÷ 6 = 7
  42 ÷ 7 = 6
```

```
  [9 ÷ 3 = 3]
* 12 ÷ 3 = 4
  12 ÷ 4 = 3
* 15 ÷ 3 = 5
  15 ÷ 5 = 3
* 18 ÷ 3 = 6
  18 ÷ 6 = 3
* 21 ÷ 3 = 7
  21 ÷ 7 = 3
* 24 ÷ 3 = 8
  24 ÷ 8 = 3
* 27 ÷ 3 = 9
  27 ÷ 9 = 3

* 48 ÷ 6 = 8
  48 ÷ 8 = 6
* 54 ÷ 6 = 9
  54 ÷ 9 = 6
```

```
 [16 ÷ 4 = 4]
* 20 ÷ 4 = 5
  20 ÷ 5 = 4
* 24 ÷ 4 = 6
  24 ÷ 6 = 4
* 28 ÷ 4 = 7
  28 ÷ 7 = 4
* 32 ÷ 4 = 8
  32 ÷ 8 = 4
* 36 ÷ 4 = 9
  36 ÷ 9 = 4

 [49 ÷ 7 = 7]
* 56 ÷ 7 = 8
  56 ÷ 8 = 7
* 63 ÷ 7 = 9
  63 ÷ 9 = 7
```

```
 [25 ÷ 5 = 5]
* 30 ÷ 5 = 6
  30 ÷ 6 = 5
* 35 ÷ 5 = 7
  35 ÷ 7 = 5
* 40 ÷ 5 = 8
  40 ÷ 8 = 5
* 45 ÷ 5 = 9
  45 ÷ 9 = 5

 [64 ÷ 8 = 8]
* 72 ÷ 8 = 9
  72 ÷ 9 = 8

 [81 ÷ 9 = 9]
```

Note: Each pair of the facts headed by "*" is related to a multiplication fact. (See p 173)
Remember: Your ability to do division depends on your knowledge of multiplication facts.

Three Ways Of Writing Division (See also "Symbols For Multiplication" p.113)

There are three ways of writing **"45 divided by 9 equals 5"** or **"9 into 45 is 5"** as seen below. ***Just Remember that*** **division can be written in fraction form.**

$$45 \div 9 = 5 \qquad \begin{array}{r} 5 \\ 9\overline{)45} \end{array} \qquad \frac{45}{9} = 5 \;\; (45/9 = 5)$$

Again, we can see that division and multiplication are inverse operations from the following demonstrations:

divisor (9)	**x**	**quotient** (5)	**=**	**dividend** (45)
(factor)		(factor)		(product)

$$\begin{tikzcd} 45 & \div & 9 \arrow[rr, bend left, "\mathrm{x}"] & = & 5 \arrow[llll, bend left, "="] \end{tikzcd}$$

$$\begin{tikzcd} & 5 \arrow[d, bend left, "="] \\ 9 \arrow[ur, bend left, "\div"] & \overline{)45} \end{tikzcd}$$

$$\begin{tikzcd} \frac{45}{9} & = & 5 \arrow[ll, bend right, "="'] \end{tikzcd} \quad \text{(9} \xrightarrow{\mathrm{x}} 5\text{)}$$

Question: Do you see that the divisor and quotient are factors? Now, if the divisor is 5, what will be the quotient?

Using The Multiplication Table To Find Quotients (Review first p.176)

Since division is an inverse operation of multiplication, **finding the quotient is the same as finding the missing factor** as shown below:

$15 \div 3 = \square$ **In division,** we are finding the quotient

$3 \times \square = 15$ **In multiplication,** we are finding the missing factor.

There are **two ways** of finding a missing factor on the multiplication table because multiplication, like addition, is **commutative:**

x	1	2	3	4	$\boxed{5}$
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
$\boxed{5}$	5	10	15	20	25

Finding The Missing Factor: $3 \times \square = 15$

* Find 3 in the top row and go down the column, stop at 15. Then go across the row to the left. The **missing factor is the number heading the row.**
* Find 3 in the first column on the left. Go across the row, stop at 15. Then go straight up to the top. **The missing factor is the number heading the column.**

The missing factor is the quotient.

Writing Quotients With Remainders (Review p.96)

The following examples show that **not every whole number can be *divided evenly without remainder:***

a)
```
        9
  3 ) 27
    - 27
    ----
       0  <--0 remainder
```

b)
```
        4
  6 ) 27
    - 24
    ----
       3  <--3 remainder
```

A remainder is a part of dividend that is left over
A remainder must always be less than the divisor: $3 < 6$.

Three Ways of Writing A Remainder: (see p.251)

1) **As A Whole Number**

```
        4 r3
  6 ) 27
    - 24
    ----
       3
```

2) **As A Fraction** (p 96)

```
        4 1/2
  6 ) 27
    - 24
    ----
       3
```

3) **As A Decimal** (p.248)

```
        4.5
  6 ) 27.0
    - 24
    ----
       3 0
       3 0
       ---
         0
```

Interpreting The Remainder (Review previous page.)

In real life, remainders are not just numbers that we write in this way or that way. We must know what the remainder means, and how to interpret it correctly in a given situation. For example,

Problem: "Mary's class is planning for a class picnic. They are going to serve hot dogs with other goodies. There will be 33 students, the teacher, and two mothers at the picnic. For each one to have one hot dog, **how many packages** of hot dogs should they buy? Hot dogs come in a package of 8 links."

Solution: 1st. Decide **the numbers of people** who will attend the picnic:

$$33 \text{ (students)} + 1 \text{ (teacher)} + 2 \text{ (mothers)} = 36$$

2nd. Divide 36 by 8 (8 links in a package):

$36 \div 8 =$ **4 with a remainder 4**

If they buy 4 packages of hot dogs, it will serve only 32 people: 4 (packages) x 8 (links) = 32
The remaining 4 people, r4, will go hungry.

Answer: For every one to have a hot dog, **they must buy 5 packages.**

Playing With Numbers (III) - Developing The Number Sense

In studying arithmetic, it is important to develop the number sense. Number sense is the **ability to see the relations that exist between numbers, and to use that existing relations to simplify the computation.** With practice, you will be able to tell which problem can be easily simplified and which one can not.

Example: Multiply 25 x 16. **(Remember: 25 x 16 = 16 x 25)**

$\boxed{4 \times 4 = 16}$

Method 1. $(25 \times 4) \times 4$

$100 \times 4 = 400$

If you remember 25 x 4 = 100 and 16 = 4 x 4, you can solve the problem right in your head.

$\boxed{100 \div 4 = 25}$

Method 2. $(16 \times 100) \div 4$

$1600 \div 4 = 400$

If you know that 25 is 1/4 of 100, you can multiply 16 by 100 and then divide the product by 4.

Example: Multiply 2.84 x 5. **(Remember: 5 = 10 ÷ 2)**

$(2.84 \times 10) \div 2$

$28.4 \div 2 = 14.2$

You can multiply the number by 10, and then divide the product by 2.

Prior Knowledge For Dividing Numbers

Division is more involved than the other three operations. However, if you can answer "yes" **with confidence to each of the following questions,** you will have little difficulty doing division.

1. **Can you give one-digit multiplication facts quickly from memory with no error? (p.173)**
2. **Can you do subtraction with borrowing fast with no error? (pp.158-160)**
3. **Do you know how to estimate? (p.119)**
4. **Do you understand place-value concept? (p.109)**

Division and Multiplication Facts: Your ability to do division depends on your knowledge of multiplication facts because division uses multiplication facts in an opposite ways (See p 193). For example,

When you divide:	**42 ÷ 6** (Inverse operations. p.198)
You think:	**"6 times *what* is 42?"** or
	"How many times 6 is contained in 42?"
You know:	**6 x 7 = 42.** So, **42 ÷ 6 = 7.**

The Skill of Estimation: You use this skill -

* to estimate the **size of the quotient** - in the tens? in the hundreds?...

* to determine the **partial quotients** - using trial quotients.

General Rule For Dividing Numbers

Division begins at the left.

```
      6
4 ) 2 5 8 0
  − 2 4
      1 8
```

Partial quotients refers to the digits that make up the quotient. **Where to place the first digit** of the quotient is very important. See next page.

Partial dividends: In dividing, we divide only a part of the dividend at a time. **The dividend becomes a series of partial dividends.** Using the above example:

* **The 1st partial dividend is 25,** since 2 cannot be divided by 4:
  (partial dividend) $25 \div 4 = 6$ **(partial quotient)** ***remainder 1***

* **The 2nd partial dividend is 18,** the remainder and the next lower digit:
  (partial dividend) $18 - 4 = 4$ **(partial quotient)** ***remainder 2***

> **Rule: In division with *whole numbers*, the dividend must be equal to or larger than the divisor** (see p.122) Therefore, a partial dividend ***must have as many digits, or one digit more*** than the divisor.

Placing The First Partial Quotient (Review "Zeros Before..." p.66)

General rule: Place the first partial quotient right above *the right-hand digit* of the partial dividend, if the partial dividend has more than one digit. Consider each of the following dividends as if they were the first partial dividend of a division problem:

a) $\begin{array}{r} 0 \\ 4\overline{)\,2} \end{array}$

2 can't be divided by 4. Write a zero above the digit 2. But zeros before whole numbers are omitted. (p.66)

b) $\begin{array}{r} 2 \\ 3\overline{)\,\mathbf{6}} \end{array}$

6 divided by 3 is 2 (3 x 2 = 6). **Write 2 above 6.**

c) $\begin{array}{r} 8 \\ 7\overline{)\,5\ \mathbf{9}} \end{array}$

56 divided by 8 is 8 r3. **Write 8 above 9, *the right-hand digit*** of the partial dividend 59

d) $\begin{array}{r} 4 \\ 1\ 2\overline{)\,4\ \mathbf{9}} \end{array}$

49 divided by 12 is 4 r1. **Write 4 above 9, *the right-hand digit*** of the partial dividend 49

e) $\begin{array}{r} \cancel{0}\ \cancel{0}\ 3 \\ 4\ 2\overline{)\,1\ 2\ \mathbf{8}} \end{array}$

42 can't go into 1, nor 12. **Write 0 above 1, & 2.**
128 divided by 42 is 3 r 2. **Write 3 above 8, *the right-hand digit*** of the partial dividend 126.
Again, **drop the two zeros that come before the whole number.**

General Process For Dividing Numbers - Long Division (Read first p.204)

In general, the process of division **follows repeatedly the sequence of steps** described below:

Step 1. ***Divide.*** **Divide the first partial dividend by the divisor.**
Write the 1st digit of the quotient *above the right-hand digit* of the partial dividend, **if the partial dividend has more than one digit.**

Step 2. ***Multiply.*** **Multiply the divisor by the 1st partial quotient (Step 1).**
Write the product under the 1st partial dividend.

Step 3. ***Subtract.*** **Subtract the product (Step 2) from the 1st partial dividend.**
Write the *remainder (or difference)* under the product.

Step 4. ***Bring down.*** **Bring down the next lower number from the dividend.**
The remainder and the number becomes the 2nd partial dividend.

Follow the same steps described above for the 2nd partial dividend.

Note: Division involving **"trial quotient"**, or **"zeros in the quotient"** does not always **follow strictly** the order of steps given above.

Things To Remember In Dividing Larger Numbers

* **Estimate The Final Quotient First** (p.119). Estimate the *approximate* size of the final quotient - will it be **in the ones? in the tens? in the hundreds?...** Give the estimate in round numbers: 10, 300, etc. Compare your estimate with the final answer.

* **The Remainder Must be Smaller Than The Divisor** (p 200). At any point in the process of division, if the remainder is **equal to or larger than** the divisor, **the next larger number** should be tried as a partial quotient.

* **Writing The Remainder In The Final Quotient** (p.200). If a division has a remainder at the end, write the quotient, then **write a small letter "r" to the right, and then write the remainder.** For example: 128 ÷ 3 = 42 r2.

  (Remember that remainder can be expressed as a fraction, a decimal, or even round-off It depends on the nature of the problem. See p.200)

* **Check The Division At The End.** Use multiplication to check division.
  a) Quotient **Without** Remainder:

  (Divisor) x (Quotient) = (Dividend)

  b) Quotient **With** Remainder:

  (Divisor) x (Quotient) + (Remainder) = (Dividend)

One-Digit Divisor - Long Division (Read first "General Process..." p.206)

(a)

$$
\begin{array}{rccc}
 & & 5 & \\ \cline{2-4}
3\,) & 1 & 7 & 4 \\
 & -1 & 5 & \downarrow \\ \cline{2-3}
 & & 2 & 4
\end{array}
$$

(a) **Steps 1. Divide.** **17 ÷ 3.** (Since 1 can't be divided by 3)
How many 3's are in 17? 5 with r2
Write 5 above 7, the right hand digit of the partial dividend 17.

2. Multiply. **3 x 5 = 15**
(The divisor) x (1st partial quotient)
Write 15 under 17, the partial dividend.

3. Subtract. **17 - 15 = 2.**
(1st partial dividend) - (The product)
Write 2, the remainder, **under 5.**

4. Bring down. Bring down 4 from the dividend.
Write 4 next to 2, the remainder.
24 becomes the 2nd partial dividend.

(b)

$$
\begin{array}{rccc}
 & & 5 & 8 \\ \cline{2-4}
3\,) & 1 & 7 & 4 \\
 & -1 & 5 & \downarrow \\ \cline{2-3}
 & & 2 & 4 \\
 & & -2 & 4 \\ \cline{3-4}
 & & & 0
\end{array}
$$

(b) **Steps 1. Divide.** **24 ÷ 3.**
Ask: How many 3's are in 24? 8
Write 8 above 4, the right hand digit of the partial dividend.

2. Multiply. **3 x 8 = 24.**
Write 24 under 24.

3. Subtract. **24 - 24 = 0.**

Let's go over the last division:

Estimate first (Read "How-To Estimate" p.119).
* Divide: 174 - 3. Think: 170 ÷ 3.
* We know: 3 x 50 = 150; and 3 x 60 = 180.
* 170 is between 150 and 180, **the quotient should be between 50 and 60.**

When 1 can't be divided by 3, **we are supposed to write 0 above 1,** but we omit it because 058 = 58 (p.66)

$$\begin{array}{r} \cancel{0}\;5\;8 \\ 3\,\overline{)\,1\;7\;4} \\ \underline{1\;5} \\ 2\;4 \\ \underline{-\;2\;4} \\ 0 \end{array}$$

17 divided by 3 is 5 with remainder 2. **It is actually 170 ÷ 3 equals 50 with remainder 20.**

3 x 5 = 15. **It is actually 3 x 50 = 150 with 0 in ones place omitted.**

Check: Always check on the work at the end.
3 x 58 = 174 (divisor x quotient = dividend)
Division should begin with estimating and end with checking.

Remember: Zeros before whole numbers are omitted. ***But** zero(s) as place holder(s) in the quotients can not, and must not, be omitted* (p.107)

One-Digit Divisor - Short Division or Invert Short Division

With a little practice, you can do division with one-digit divisor (even simple two-digit divisor) **mentally.** The method is called **short division** (a) or **invert short division** (b). For example, let's divide 8706 ÷ 6:

(a)

```
    1 4 5 1
6 ) 8 7 0 6
     2 3
```

Begin at the left the thousands place.

a. **6 divided by 8 = 1. Write 1 above (or below) 8,** carry the remainder 2. (8 - 6 = 2)

Before you get used to the method, you may want to jot down the remainder, as shown, to remind you.

or

(b)

```
6 ) 8 7 0 6
    -------
    1 4 5 1
```

b. **27 divided by 6 = 4. Write 4 above (or below) 7,** carry the remainder 3. (27 - 24 = 3)

c. **30 divided by 6 =** 5. **Write 5 above (or below) 0,** carry the remainder 0. (30 - 30 = 0)

d. **6 divided by 6 = 1. Write 1 above (or below) 6.**

Example (b) is called invert short division because **the division symbol is written upside down with the quotient under the dividend.**

Connection: When studying *fractions*, you need this skill **in prime factorization.** **(p.297)**

Division With Numbers Ending In Zeros (Review first "Dividing By 10..." p.179)

a)

$$\begin{array}{r} 4 \\ 20\,\overline{)\,80} \\ -\,80 \\ \hline 0 \end{array}$$

* **Divide.** 8 divided by 2 is 4. 80 divided by 20 is 4.
  Write the quotient 4 above 0.
* **Multiply.** 20 x 4 = 80. Write 80 under 80.
* **Subtract.** 80 - 80 = 0.

$$\begin{array}{r} 4 \\ 2\cancel{0}\,\overline{)\,8\cancel{0}} \end{array}$$

Shortcut: Cancel the zero *in both* the divisor and dividend. Then divide.

b)

$$\begin{array}{r} 60 \\ 40\,\overline{)\,2400} \\ -\,240 \\ \hline 00 \end{array}$$

* **Divide.** 24 divided by 4 is 6. 240 divided by 40 is 6.
  Write the first digit of the quotient in tens place above 0.
* **Multiply.** 40 x 6 = 240.
* **Subtract.** 240 - 240 = 0. Add 0 to 6.

$$\begin{array}{r} 60 \\ 4\cancel{0}\,\overline{)\,240\cancel{0}} \end{array}$$

Shortcut: Cancel the zero *in both* the divisor and dividend. Then divide.

Rule: You can cancel *the same number of zeros in both* the divisor and dividend.
You can cancel only the zero(s) that are at the end of the numbers.

Zeros In The Quotient (Read first "General Procedure..." p.206)

Study carefully the following two examples and **notice how zeros in the quotient occur:**

```
     2 0
4 ) 8 2 4
  - 8 ↓
      2
```

Estimate:

* Think 4 x ( ) = 800. 4 x 200 = 800.
The quotient should be about 200.

Steps **1.** 8 divided by 4 = 2. Write 2 above 8.
2. Multiply 4 by 2 = 8. Write 8 under 8.
3. Subtract 8 from 8 = 0. Write nothing.
4. ***Bring down*** 2 from the dividend.
2 can not be divided by 4, *write 0 above 2.*
0 is a place holder
4. ***Bring down*** 4 from the dividend.
24 becomes the 2nd partial dividend.

```
     2 0 6
4 ) 8 2 4
  - 8 ↓ ↓
      2 4
    - 2 4
        0
```

Steps **1.** 24 divided by 4 = 6. Write 6 above 4.
2. Multiply 4 by 6 = 24. Write 24 under 24.
3. Subtract 24 - 24 = 0.

Note: If it were 82 - 4, the quotient would have been 20 with r2.

Zeros In The Quotient

```
          3 0 9
 2 2 ) 6 7 9 8
      - 6 6 ↓ ↓
          1 9 8
         -1 9 8
              0
```

Check:

```
        3 0 9
      x   2 2
        6 1 8
    + 6 1 8 ↓
      6 7 9 8
```

Estimate:

* Think: 6000 ÷ 20 or 600 ÷ 2 = 300.

* The quotient should be about 300.

Steps
1. 22 into 67 = 3r1 Write 3 above 7.
2. 22 x 3 = 66. Write 66 under 67
3. 67 - 66 = 1 Write 1 under 6
4. ***Bring down 9.*** Write 9 next to 1.
19 < 22 ***Write 0 above 9***
4. ***Bring down 8.*** Write 8 next to 9.
198 is the 2nd partial dividend.

Steps
1. 22 into 198 = 9. Write 9 above 8.
2. 22 x 9 = 198. Write 198 under 198.
3. 198 - 198 = 0. Write 0 under 8.

Zeros In The Quotient occur when the partial dividend is less than the divisor, and you have to bring down numbers from the dividend for **two consecutive times** - one after another.

Two-Digit Divisor - Trial Quotients (Review first "General Process..." p.206)

When divided by two-digit or larger divisors, you may have to start out with **"trial quotients"** by estimating. Example, divide 1102 - 38:

Trial quotient

```
            3
3 8 ) 1 1 0 2
    - 1 1 4
```

Trial: * Divide 110 - 38.
Think 11 divided by 3 = 3 r2. Try 3.
* Multiply 38 x 3 = 114.
* **Compare 114 with 110: 114 > 110.**
3 is too big. **Try 2.**

```
            2 9
3 8 ) 1 1 0 2
      - 7 6
        3 4 2
      - 3 4 2
            0
```

Steps **1.** Write 2 above 0.
2. Multiply 38 x 2 = 76.
3. Write 76 under 10 and subtract.
4. Bring down 2.

Steps **1.** Divide 342 - 38. **Think 36 ÷ 4 = 9**
Write 9 above 2.
2. Multiply 38 x 9 = 342.
3. Write 342 under 342 and subtract.

Note: You know the trial quotient (estimate) is too large, if you can't subtract; and too small when the remainder (difference) is larger than the divisor

Trial Quotient: If we have noticed that **38 is close to 40,** we would have used 40 as a trial divisor. 110 ÷ 40 would be **2 instead of 3.** ***To keep your work neat and clean, write on a scratch paper while computing trial quotient.***

$$38\,\overline{)\,1\,1\,\mathbf{0}\,2}$$

110 can be divided by 38, but not 11. A partial dividend must be **equal to or larger than** the divisor. **(Please read p.122)**

$$\begin{array}{r} 2 \\ 38\,\overline{)\,1\,1\,\mathbf{0}\,2} \end{array}$$

The first digit of the quotient (2) must be **above the right-hand digit** of its partial dividend (110).

$$\begin{array}{r} \text{tens} \rightarrow 2 \\ 38\,\overline{)\,1\,1\,0\,2} \\ \underline{-\,7\,6} \end{array}$$

76 = 38 x 2. **Actually 38 x 20 = 760 with 0 in the ones place omitted.** To place 76 under 11 instead of 10 would make 76 equals to 7600, 10 times larger.

Remember: Divisions with "Trial Quotients" require ***the skill of estimating*** and ***some guessing.*** With practice, you will be able to recognize the correct partial quotient without having to go through several trial quotients.

Three-Digit Divisor - Trial Quotient (Review first pp.206-207)

Estimate: Think: 90,000 ÷ 300 = 300. The quotient should be about 300.

```
            2
2 7 3 ) 8 5 1 7 6
      - 5 4 8
        3 0 3
```

Trial: * 851 divided by 273. **Think: 8 divided by 3 = 2 r2**
* **Try 2.** 274 x 2 = 548.
* 851 - 548 = 303. **Compare: 303 > 273.**
2 is too small, **try 3.**

```
            3 1 2
2 7 3 ) 8 5 1 7 6
      - 8 1 9 ↓ |
          3 2 7 |
        - 2 7 3 ↓
            5 4 6
            5 4 6
                0
```

Steps **1.** 273 into 851 = 3 r32. Write 3 above 1.
2. 273 x 3 = 819. Write 819 under 851.
3. 851 - 819 = 32. Write 32 under 19.
4. Bring down 7. Write 7 next to 32.

Steps **1.** 273 into 327 = 1 r54 Write 1 above 7.
2. 273 x 1 = 273. Write 273 under 327.
3. 327 - 273 = 54. Write 54 under 73.
4. Bring down 6. Write 6 next to 54.

Steps **1.** 273 into 546 = 2. Write 2 above 6.
2. 273 x 2 = 546. Write 546 under 546.
3. 546 - 546 = 0.

Finding Averages

Problem: On a recent family vacation, the Smiths kept the record of the distance the family travelled each day as follows:

First day - 300 miles Second day - 250 miles
Third day - 300 miles Fourth day - 350 miles
Fifth day - 200 miles

And you want to know the **average distance** the Smiths family travelled a day during the entire trip.

Solution: Step 1. Adding together the numbers of miles covered in 5 days:

① ② ③ ④ ⑤ ← Numbers of Addend
300 + 250 + 300 + 350 + 200 = 1,400 miles ← Sum

Step 2. Divide the sum by the number of addends: (here the total number of days)

$1{,}400 \div 5 = 280$ miles ← **Average**

Answer: The Smiths family travelled **on an average of 280 miles a day.**

To Find Averages: Add the numbers together and then **divide** the sum by the number of addends.

Order Of Operations (Combined Operations)

We know how to work with addition, subtraction, multiplication, and division separately. But what should we do when **more than one operations** are used in one number sentence? For example:

(a) Is $20 + 5 \times 4 = 100$ or 40?
(b) Is $(20 + 5) \times 4$ the same as $20 + 5 \times 4$?
(c) Is $40 \div 8 \times 5 = 25$ or 1?
(d) $[12 - (2 \times 3)] - 6 = ?$
(e) $4 + 9 \div 3 \times 2^2 = ?$

To prevent different answers to the same arithmetic problems, the general rule of procedure has been internationally accepted in working with combined operations. The rule of procedure is given below:

Order Of Operations:

1st. Work inside the grouping symbols. When there are more than one grouping symbols as in example (d), work from the inside out.
2nd. Work on the exponents as in example (e).
3rd. Do all the multiplication and division, from left to right, in the order in which they appear.
4th. Do all the addition and subtraction, from left to right, in the order in which they appear.

Examples Of Order Of Operations

Let's apply the rule to the examples given on the previous page. We assign numbers ①②③ above the operation symbols **to indicate the order of operations:**

(a) $20 \overset{\textcircled{2}}{+} 5 \overset{\textcircled{1}}{\times} 4 \longrightarrow 20 + 20 = 40$ (not 100)

(b) $(20 \overset{\textcircled{1}}{+} 5) \overset{\textcircled{2}}{\times} 4 \longrightarrow 25 \times 4 = 100$ (not 40)

(c) $40 \overset{\textcircled{1}}{\div} 8 \overset{\textcircled{2}}{\times} 5 \longrightarrow 5 \times 5 = 25$ (not 1)

(d) $[12 \overset{\textcircled{2}}{-} (2 \overset{\textcircled{1}}{\times} 3)] \overset{\textcircled{3}}{-} 6 \longrightarrow [12 \overset{\textcircled{2}}{-} 6] \overset{\textcircled{3}}{\div} 6 \longrightarrow 6 - 6 = 1$

(e) $4 \overset{\textcircled{4}}{+} 9 \overset{\textcircled{2}}{-} 3 \overset{\textcircled{3}}{\times} \overset{\textcircled{1}}{2^2} \longrightarrow 4 \overset{\textcircled{4}}{+} 9 \overset{\textcircled{2}}{-} 3 \overset{\textcircled{3}}{\times} 4 \longrightarrow 4 \overset{\textcircled{4}}{+} 3 \overset{\textcircled{3}}{\times} 4 \longrightarrow 4 + 12 = 16$

(f) $48 \overset{\textcircled{1}}{-} 6 \overset{\textcircled{2}}{\times} 3 \overset{\textcircled{4}}{-} 4 \overset{\textcircled{3}}{\times} 6 \overset{\textcircled{5}}{+} 5$ Can you try this one? The answer is 5.

Summary
(Division)

* Division is a repeated subtraction and an inverse operation of multiplication. The division properties are also related to that of multiplication.
* The division facts are the reverse of multiplication facts. The quotient and the divisor are the factors, and the dividend is the product. Because of this, we can use the multiplication table to find the quotient, a missing factor
* There are three ways of writing division. Fraction form is one of them. Since division by zero is impossible, fractions can not have "0" as the denominator
* The remainders in quotients can be written as a whole number, a decimal, or a fraction. However, in real life, we either drop the remainder or increase the quotient by 1 depending on the situation.
* Division, unlike the other three operations, begins at the left. In general, division follows repeatedly 4 steps divide, multiply, subtract, and bring down. If the partial dividend has more than one digit, the partial quotients are always placed right above the right-hand digit of the partial dividend.
* When more than one operations are present in a number sentence, follow the rule on "order of operations" in your computation.