the family MATH companion

ARITHMETIC—THE FOUNDATION OF MATH

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Part I. Numbers & Concepts

A. Introduction

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Different Sets of Numbers

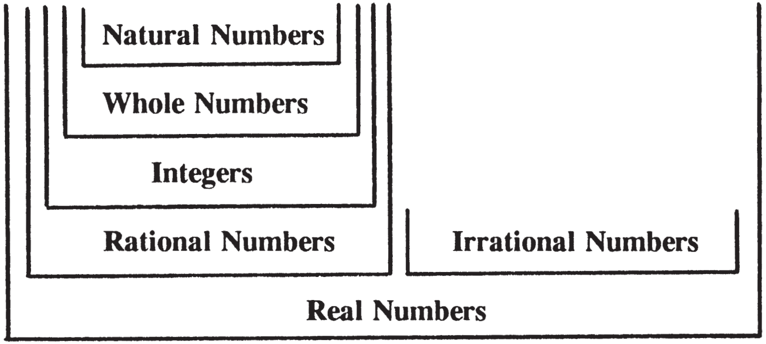
A set is a collection of objects, numbers, etc. that go together. The following are different sets of numbers that we use in mathematics:

(1)	**1, 2, 3, ...**	**Natural Numbers**	**Rational Numbers**
(2)	**0, 1, 2, 3, ...**	**Whole Numbers**	
(3)	**... -3, -2, -1, 0, 1, 2, 3, ...**	**Integers**	
(4)	**... -3.5, - 2.25, 0.125 0.33, ...**	**Decimals**	
(5)	**... -1/8, -2/3, 4/1, 38/100, ...**	**Fractions**	
(6)	**π ("pi"), $\sqrt{2}$, $\sqrt{5}$, ...**	**Irrational Numbers**	
(7)	**Rational numbers & Irrational numbers**	**Real Numbers**	

Note: The three dots means the numbers continue without end.

Sets of Numbers And Their Relations

- The set of **natural numbers** (1, 2, 3,...) ***is contained in*** the set of **whole numbers** (0, 1, 2, 3,...). (Natural numbers are also called counting numbers.)
- The set of **whole numbers** (0, 1, 2, 3,...) ***is contained in*** a larger set of **integers** (...,-3, -2, -1, 0, 1, 2, 3,...).
- The set of **integers** ***is contained in yet*** a larger set of **rational numbers** (...,-1/2, -2/5, 3/1,...).
- The set of **rational numbers** and the set of **irrational numbers** ***make up*** the set of **real numbers** as illustrated in the following diagram:



If a square root of a positive integer is not an integer, the square root is irrational

Connection: Knowing the different sets of numbers, their relations and their distinctions, will help you in your study of math later on.

Relations Among Sets Of Positive Numbers

In arithmetic, we deal with only three sets of positive numbers:

(1) **Whole Numbers:** 0, 1, 2, 3, ...
(2) **Decimals:** 0.5, 0.15, 0.33, ...
(3) **Fractions:** 1/2, 1/10, 35/100, ...

Strictly speaking, "0" is neither positive nor negative.

The following show the relations among these three sets of numbers:

1. **All three sets of numbers are part of rational numbers** - a number that can be written in the form a/b (fraction).

2. **Any whole number can be written as a fraction (a/b)** - a knowledge needed to work with fractions **(See p.298)**. But, there is a distinction between "fractions" and "whole numbers" as rational numbers:
   Example: A fraction 1/3 is a fraction, and ***can never be*** a whole number.
   Example: A whole number 7 equals 7/1, a fractional number, ***but*** 7 ***is not*** a fraction.

3. **Any decimal can be written as a fraction.** In fact, decimals and fractions are two different ways of writing the same number **(See p.98)**.

Classification Of Whole Numbers (1) - Even & Odd Numbers

We divide the set of whole numbers into **two major groups** called even and odd numbers. Their differences can be described in the following ways:

* The whole numbers **alternate** between even and odd:

The **even** numbers: 0 2 4 6 8 10 ...
The **odd** numbers: 1 3 5 7 9 ...

The set of whole numbers are ***either*** even ***or*** odd, and **no number is *both*** even and odd. **Zero is considered as an even number.**

* The even numbers can be **divided evenly by 2,** and the odd numbers **can not:**

Even	**Odd**
2 - 2 = 1	3 - 2 = 1 r1
4 ÷ 2 = 2	5 ÷ 2 = 2 r1
6 - 2 = 3	7 ÷ 2 = 3 r1

All even numbers are multiples of 2.

Any number ending in 2, 4, 6, 8, & 0 has 2 as a factor (See p.288).

* The even numbers can always be **grouped by 2,** but the odd numbers always have **1 left over.**

Classification Of Whole Numbers (2) - Prime & Composite Numbers (see p.290).

The set of whole numbers, **except 0 and 1,** can also be classified as prime and composite numbers. For an illustration, let's list the factors of 3, 5, 9, 15, and 21. **Factors are exact divisors** (see p.282).

2 factors	2 factors	3 factors	4 factors	4 factors
3 ÷ 1 = 3	5 ÷ 1 = 5	9 - 1 = 9	15 ÷ 1 = 15	21 ÷ 1 = 21
3 ÷ 3 = 1	5 ÷ 5 = 1	9 - 3 = 3	15 ÷ 3 = 5	21 ÷ 3 = 7
		9 ÷ 9 = 1	15 ÷ 5 = 3	21 ÷ 7 = 3
			15 ÷ 15 = 1	21 ÷ 21 = 1

prime numbers

- * **Prime Numbers** are the numbers that have *only two factors:* 1 and itself.
- * **Composite Numbers** are the numbers that have *more than two factors.*

Prime Numbers vs. Odd Numbers - It is important not to confuse prime numbers with odd numbers. In the above example, 3, 5, 9, 15, 21, are all odd numbers, **but only 3 and 5 are prime numbers.**

Connection: The knowledge of prime and composite numbers is needed in working with fractions.

Cardinal Numbers & Ordinal Numbers

Cardinal Numbers are numbers that are used **to count** or to tell how many.
Example: 1, 2, 5, 8, 21, ...

Ordinal Numbers are numbers that are used to tell, for example,
- **the position in a contest:** first place, second place, etc....
- **the succession of something in a series:** first day of the week, etc....

Writing Ordinal Numbers:

The following shows how to write ordinal numbers:
* First (1st), Second (2nd), Third (3rd), Fourth (4th), Fifth (5th), Sixth (6th), Seventh (7th), Eighth (8th), Ninth (9th), Tenth (10th).
* Eleventh (11th), Twelfth (12th), Thirteenth (13th), Fourteenth (14th), Fifteenth (15th), Sixteenth (16th), Seventeenth (17th), Eighteenth (18th), Nineteenth (19th), Twentieth (20th)
* Twenty-first (21st), Twenty-second (22nd), Twenty-third (23rd), Twenty-fourth (24th),...
* Thirty-first (31st), Thirty-second (32nd) , Thirty-third (33th),...
* Ninety-first (91st),... hundredth (100th).

Connection: We use ordinal numbers in reading exponents **(See p.42)**.

Summary

(Introduction)

- Arithmetic deals only with three positive numbers - whole numbers, decimals, and fractions.
- A rational number is a number that can be written as a quotient of two whole numbers a/b, with 0 excluded as the denominator
- Any whole number can be written as a fraction, but a fraction can not be written as a whole number, except the fractions with 1 as the denominator
- Decimals and fractions are two different ways of writing the same number
- The set of whole numbers can be classified as "even and odd numbers" or as "prime and composite numbers" - with 0 and 1 excluded.

Part I. Numbers & Concepts

B. Whole Numbers

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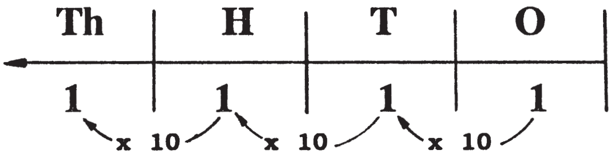
Decimal System Or Base-10 System (See also p.34)

Our number system is called "Decimal System" ("deci" means "ten") or "Base Ten System" because it is **based on 10.** In decimal system, each digit is **10 times larger than the digit to its right** and only **one tenth (1/10) of the digit to its left.** The following shows how decimal system works:

O - Ones; T - Tens
H - Hundreds; Th - Thousands



The number **1,111**

10 is 10 times larger than 1, the digit to **its right.**
(10 = 10 ones) And

10 is only 1/10 of 100, the digit to **its left.**

100 is 10 times larger than 10, the digit to **its right.**
(100 = 10 tens) And

100 is only 1/10 of 1,000, the digit to **its left.**

Decimal System - A Place Value System (See also p.109)

The decimal system is also a place value (or positional) system, which means the value of a digit depends on **its place or position in the number.** For example, the number 5,555 has the **face value** of 5, 5, 5, 5, but the **place value** of each "5" is different, depending on its position in the number as seen below:

O - Ones; T - Tens
H - Hundreds; Th - Thousands

Th	H	T	O	
5	5	5	5	The number 5,555
			5	the 1st 5 = **5** (5 **ones**)
		5	0	the 2nd 5 = **50** (**10 times larger** than 1st "5")
	5	0	0	the 3rd 5 = **500** (**10 times larger** than 2nd "5")
5	0	0	0	the 4th 5 = **5000** (**10 times larger** than 3rd "5")

Note: Add 1 zero: 5 becomes **50** (= 5 x **10**), and so on (See p.178).

Chart: Place Value Of Whole Numbers

To make it easier to read and write large numbers, we separate the numbers by **"commas"** into groups of **three** digits called **"periods"** beginning at the right as demonstrated below:

Period 4 **Billions**			Period 3 **Millions**			Period 2 **Thousands**			Period 1 **Ones**		
Hundreds	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**
	2	**1**	**7**	**5**	**0**	**1**	**0**	**9**	**4**	**6**	**8**
③	②	①	③	②	①	③	②	①	③	②	①

In writing whole numbers, we omit the decimal point.

counting from right

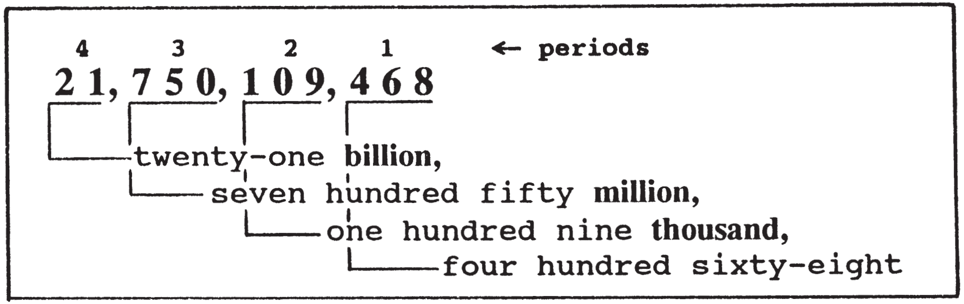
Study the chart until it is firmly fixed in your mind. You need the knowledge in reading and writing large numbers and in arithmetic operations.

Translating Numbers In Numerical Form Into Words

Example: Write 21,750,109,468 in word form.

Step 1. Determine the **largest period** of the number.

The number is in the billions. (4th period)



Step 2. **Start from left,** read or write each group of **three digits** in the normal way followed by the **period name** and **a comma.** Repeat this until you reach the ones period. The word name for 21,750,109,468 is

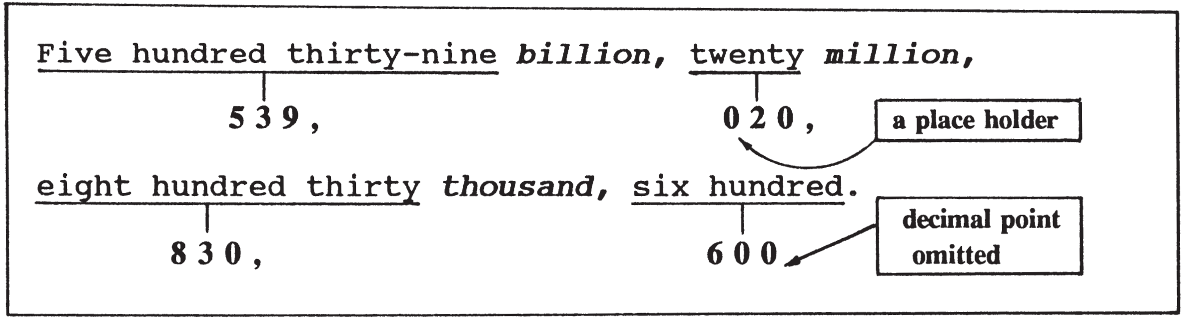
"twenty-one *billion*, seven hundred fifty *million*, one hundred nine *thousand*, four hundred sixty-eight".

Translating Numbers In Words Into Numerical (Standard) Form (1) (Review p.18)

Example: Write "five hundred thirty-nine billion, twenty million, eight hundred thirty thousand, six hundred" in numerical form.

Step 1. Remember that numbers are **grouped in three digits with a comma.**

Step 2. Read the number and translate the word name into digits and **place a "comma" where the period name occurs** (billion, million, thousand). **Use zeros as place holders in all vacant places.**



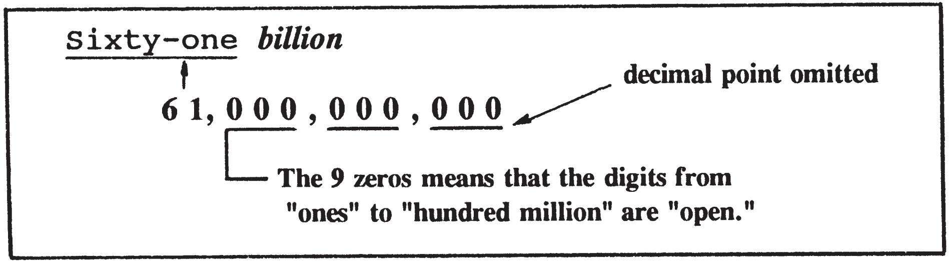
The numerical form of the number is **539,020,830,600.**

Translating Numbers In Words Into Numerical (Standard) Form (2)

Example: Write "sixty-one billion" in numerical (or standard) form.

Step 1. Remember that numbers are **grouped in three digits with a comma.**

Step 2. **Use "0" as a "place holder"** so that the digit "6" will be in the "billions" place and not in "ones" or other place.



The numerical form of the number is **61,000,000,000**

Remember:

one thousand	-	1,000	-	3 zeros
one million	-	1,000,000	-	6 zeros
one billion	-	1,000,000,000	-	9 zeros

Things To Remember In Translating Numbers

To translate numbers in numerical form into words:

* The name of the first period "ones" is not named.
* Write the period names in singular **(without "s")** instead of plural.
* The word **"and"** is **never used** in naming/reading **whole numbers.**
* Use a "hyphen" for two-digit numbers between 21 and 99, except 30, 40, 50, 60, 70, 80, 90.

To translate numbers in words into numerical form:

* **Each period must have 3 places, so use zero(s) as place holder(s), if necessary.**
* The comma is optional for 4-digit numbers. For example, 5000 and 5,000 are both acceptable.

To read 4-digit numbers:

* Translate the 4-digit numbers into the fewest possible words. For example, 1503 can be read "fifteen hundred three," instead of "one thousand five hundred three."

Connection: Every whole number has a decimal point at the end of ones (units) digit, which is omitted. You need to know this in writing "scientific notation," doing "long division," writing "money," etc.

Mastering The Concept Of Place Value & Decimal System

If you want to do well in Math, you need to understand our **number system - the decimal system** and **the place value system.** For example, If you know the concept of **place value** well, you will be able to do the following works with little difficulty:

- Reading & writing large numbers
- Writing numbers in expanded form
- Writing numbers in standard form
- Comparing & Ordering
- Rounding
- Estimating
- Adding
- Subtracting
- Multiplying
- Dividing
- Regrouping - Carrying & Borrowing
- Decimals & Fractions
- Money, and so on.

And, if you understand **decimal system** well, you will find it easy to work with powers of 10, multiples of 10, scientific notation, etc.

Writing Numbers In Expanded Form (Review first "Place Value Chart" p.18)

To write a number in expanded form is to write the number as the **sum (addition) of the value of each digit.** The following shows how to write 2073 in expanded form.

Step 1. Find the value of each digit.

To find the value of a digit, multiply the digit by its Place Value.

2, 0 7 3

3 x **1** (place value) = **3**
7 x **10** (place value) = **70**
0 x **100** (place value) = **0**
2 x **1000** (place value) = **2000**

The value of each digit.

Step 2. Write the number as the **sum** of the value of each digit.

standard numeral → **2,073 = 2,000 + 70 + 3** ← expanded form

The zero (0) digit in standard form is omitted in expanded form.

Writing Standard Numbers In Exponential Notation (Read pp.36,45)

We can write numbers in expanded form or in exponential notation because **the place value of each digit is a power of 10 (See "Decimal System" p.16)** To write in exponential notation, first write the number in expanded form and then in exponents. The following shows how to write 9357 in exponential notation:

9357	**Write the number 9375 in expanded form:**
9000 + 300 + 50 + 7	**As the sum of the values of the digits.**
$9(1000) + 3(100) + 5(10) + 7(1)$	**As the sum of the products of each digit and its place value (powers of 10).**
$9(10^3) + 3(10^2) + 5(10^1) + 7(10^0)$	**Then write the powers of 10 in exponent form.**

Note: If you know the **place value** and **powers of 10**, you can write directly.

Writing Numbers in Expanded Form as Standard Form (1)

There are two ways to write 800,000 + 50,000 + 6,000 + 300 + 1 in standard form.

Method 1. Column Addition

First, line up the numbers vertically according to their place value beginning from right, from the smallest number to the largest number. Then, add. **The sum is the standard numeral.**

	place value
1	ones
00	tens
300	hundreds
6,000	thousands
50,000	ten-thousands
+ 800,000	hundred-thousands
856,301	

Standard numeral

According to the decimal system, each number should have one more zero (10 times larger) than the number right above it.

The place for the "tens" digit is listed above, on purpose, to show that "zero digit" (no tens) is omitted in the expanded form, but it is included in the standard numeral as a place holder.

Writing Numbers in Expanded Form as Standard Forms (2)

Use the following method to write 800,000 + 50,000 + 6,000 + 300 + 1 in standard form if you know the place value well.

Method 2. Direct Method

1st. Start from left to right, from the largest number to the smallest, write the "nonzero-digits" (8, 5, 6, 3, 1) together in decreasing order according to their place values. Use "0" for "vacant" digit if necessary. Remember **each period contains three digits.**

2nd. Check to see if the standard numeral has as many digits as there are in the largest-place-value-digit (the first number from the left).

largest-place-value-digit: **800,000** ← 6 digits
standard numeral: **856,301** ← 6 digits

Zero (0) holds the "tens" place so that the other digits, 3, 6, 5, 8 can be in their proper places.

Connection: Writing numbers in expanded form for arithmetic operations can help beginners to understand how operations work involving regrouping such as carrying and borrowing. **(See pp.139, 159)**

Symbols for Comparing Numbers (Including Decimals, Fractions): > & <

Two of the symbols used in comparing and ordering numbers are > and < as seen in the following examples:

	Compare:			**Read:**
(lesser number)	$3 < 4$	(larger number)	--	3 **is less than** 4.
(larger number)	$4 > 3$	(lesser number)	--	4 **is greater than** 3.

< means **"is less than"** > means **"is greater than"**

Remember:

* The symbol **is always pointed to the lesser** of the two numbers.
* In reading comparison, always read from **left to right.**
* In ordering numbers, **the symbols must point in the same direction.**

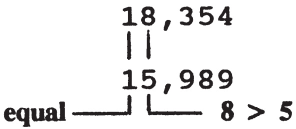
Other Symbols used in comparing numbers are:

= means "is equal to"
≠ means "is not equal to"
≥ means "is equal to or greater than"
≤ means "is equal to or less than"

Comparing and Ordering Whole Numbers (Review first "Place Value" p.18)

The easiest way is to line up the numbers according to their place values. Then **compare digits of the same place value beginning from the left.**

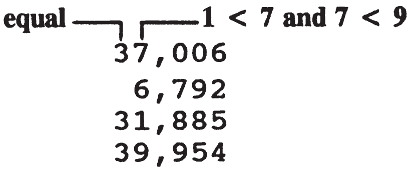
a) **Comparing Two Numbers:** 18,354 & 15,989



Begin from left, **find the first pair of digits that are not equal.** Since 8 > 5, 18,354 > 15,989 or 15,989 < 18,354

b) **Comparing & Ordering More Than Two Numbers:** 37,006, 6,792, 31,885, 39,954

General rule for whole numbers: *The fewer* the digits, *the smaller* the number; *the more* the digits, *the greater* the number.



Compare two numbers at a time **first, then order** them from least to greatest:

6,792 < 31,885 < 37.006 < 39,954

(The symbols must point in the same direction.)

Note: To order from largest to least, just **reverse the order.**

Rounding Off (Review first "Place Value Chart" p.18)

Rounding off a number is to express the number as an **approximation** to a required place (nearest ten, hundred, etc.), the place depending on the **degree of precision desired.** The following shows how to round 7548 and 9152 to the nearest hundred, respectively.

(a) $7\underline{5}48$

(b) $9\underline{1}52$

1. **Mark off the digit** to which the number is to be rounded with an underline or any mark of your choice.

Underline the digit "hundred" (the 3rd digit from the right).

4 is less than 5, so the digits to its right, 48, is rounded down to 00. The rounded number, 7500, is smaller than the actual number.

(a) $$7\underline{5}48 \\ \downarrow\downarrow \\ 7500$$

2. If the digit *to its right* is **less than 5** (4, 3, 2, 1), replace all the digits to its right with zeros.

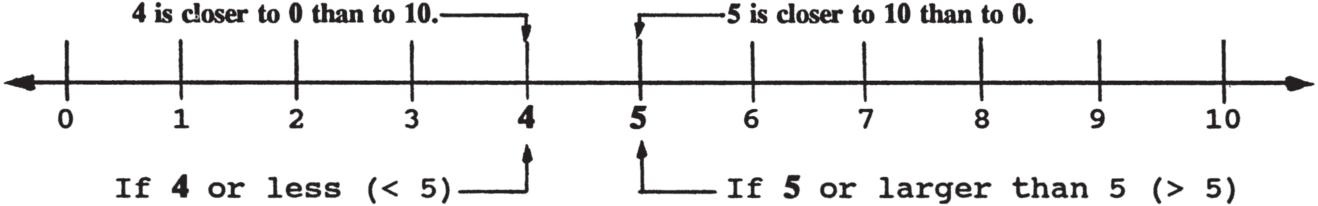
7548 is rounded to the nearest hundred.

5 or greater than 5 (> 5), so 52 was rounded up to 100. ("+1" = + 100)
The rounded number, 9200, is larger than the actual number.

(b) $\overset{+1}{9\underline{1}52} \rightarrow 9200$

3. If the digit *to its right* is **5 or greater than 5** (5, 6, 7, 8, 9), **add 1** to the underlined digit and replace all the digits to its right with zeros.

Demonstrating The Rules On A Number Line:



4 is closer to 0 than to 10.

5 is closer to 10 than to 0.

If **4** or less (< 5)
Rounding **down** to 0

If **5** or larger than 5 (> 5)
Rounding **up** to 10 (+ 1)

The rounded number will be less than the actual number.

The rounded number will be larger than the actual number.

Rounding Up or Rounding Down

Study the following examples carefully and note the result of rounding a number up or down. Will it affect estimation? Think!

Example: Round 2,499 and 1,500 to the nearest thousand, respectively.

$$\begin{array}{cccc} & \downarrow & & \\ \underline{2}, & 4 & 9 & 9 \\ & \downarrow & \downarrow & \downarrow \\ \underline{2}, & 0 & 0 & 0 \end{array}$$

Rounding Down:

Since 4 < 5, 499 was rounded down to 000.
The rounded number is **less than** the actual number.

When rounding to the nearest thousand, 2,499 is the largest number and 1,500 is the least numbers that are rounded to 2,000.

$$\begin{array}{cccc} +1 & \downarrow & & \\ \underline{1}, & 5 & 0 & 0 \\ & \downarrow & \downarrow & \downarrow \\ \underline{2}, & 0 & 0 & 0 \end{array}$$

Rounding Up:

Since 5 = 5, 500 was rounded up to 1000: +1.
The rounded number is **more than** the actual number.

Connection: Learn the skill of rounding, it is one of the methods used in **estimating** sums, differences, products, quotients, etc.

Rounding to various places - A number can be rounded to different places depending on how close to the actual value your estimate needed to be. For example, let's round 62,594 to:

(a) the nearest **ten thousand:**

$$\begin{array}{cccccc} & \downarrow & & & & \\ \underline{6} & 2 & , & 5 & 9 & 4 \\ & \downarrow & & \downarrow & \downarrow & \downarrow \\ 6 & 0 & , & 0 & 0 & 0 \end{array}$$

(b) the nearest **thousand:**

$$\begin{array}{cccccc} & +1 & & \downarrow & & \\ 6 & \underline{2} & , & 5 & 9 & 4 \\ & & & \downarrow & \downarrow & \downarrow \\ 6 & 3 & , & 0 & 0 & 0 \end{array}$$

When a number is rounded to the **largest** or **larger place,** it gives a "**rough**" estimate. The rounded number will be far over or under the actual value depending on whether the number is rounded up or rounded down. (a) & (b)

(c) the nearest **hundred:**

$$\begin{array}{cccccc} & & & +1 & \downarrow & \\ 6 & 2 & , & \underline{5} & 9 & 4 \\ & & & & \downarrow & \downarrow \\ 6 & 2 & , & 6 & 0 & 0 \end{array}$$

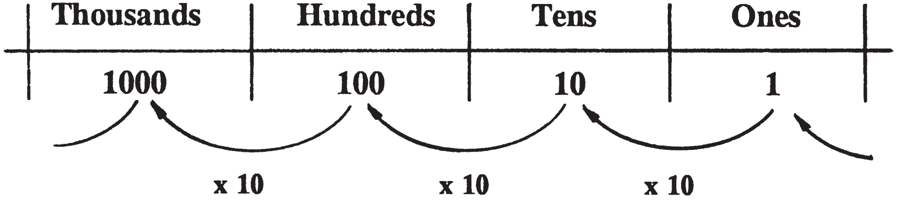
(d) the nearest **ten:**

$$\begin{array}{cccccc} & & & & & \downarrow \\ 6 & 2 & , & 5 & \underline{9} & 4 \\ & & & & & \downarrow \\ 6 & 2 & , & 5 & 9 & 0 \end{array}$$

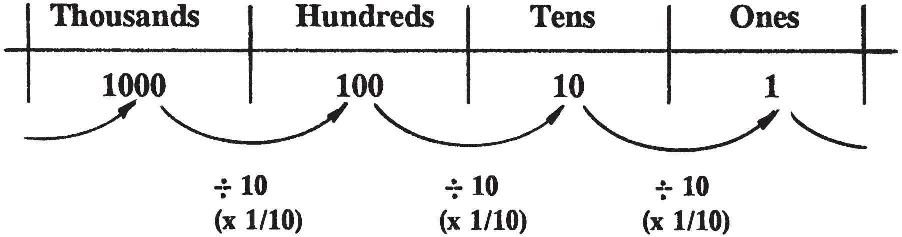
When a number is rounded to a **lesser place,** the rounded number will be **closer** to the actual value whether the number is rounded up or rounded down. (c) & (d)

Another Look At The Decimal System (See also p.16)

The following gives you another look at how our decimal system works. It can be extended in both directions without end. Moving ***to the left,*** the numbers become **larger and larger.** Moving ***to the right,*** the numbers become **smaller and smaller** and eventually **leads to decimals** - numbers less than 1 in value.



Add 1 zero when multiplying by 10.



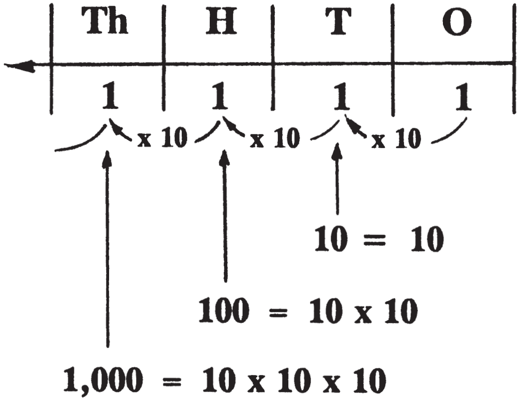
Drop 1 zero when dividing by 10.

To divide by 10 is the same as to multiply by its reciprocal, 1/10.

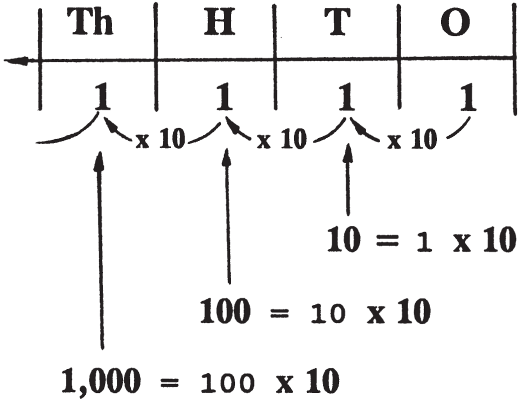
Decimal System, Powers Of 10, & Multiples Of 10

Since our number system is based on 10, the **value of each place** can be expressed in "powers of 10" (multiplication of 10s) or in "multiples of 10" (10 multiplied by any number) as shown below:

In Powers of 10:



In Multiples of 10:



Note: Study carefully the charts found on the next two pages. You will find them useful in working with **powers of 10 (exponents)** and **multiples of 10**, etc.

Chart: Expressing Place Value In Various Forms

The following two tables show that **the place value of our decimal system** can be written in various forms.

In Word Name	In Standard Form	In Multiples of 10 (x 10)	In Exponents
ones	1	= 1	= 10^0
tens	10	= 1 **x 10**	= 10^1
hundreds	100	= 10 **x 10**	= 10^2
one thousands	1,000	= 100 **x 10**	= 10^3
ten thousands	10,000	= 1,000 **x 10**	= 10^4
hundred thousands	100,000	= 10,000 **x 10**	= 10^5
one millions	1,000,000	= 100,000 **x 10**	= 10^6
ten millions	10,000,000	= 1,000,000 **x 10**	= 10^7
hundred millions	100,000,000	= 10,000,000 **x 10**	= 10^8
one billions	1,000,000,000	= 100,000,000 **x 10**	= 10^9
ten billions	10,000,000,000	= 1,000,000,000 **x 10**	= 10^{10}
hundred billions	100,000,000,000	= 10,000,000,000 **x 10**	= 10^{11}

Note: The number of zeros in Standard Form = the exponent number.

Chart: Expressing Place Value In Various Forms (See also pp.44,45)

The powers of 10, the basis of our decimal system, can be expressed in "exponent form," "factor form," and "standard form." (See also p.42)

For zero power & first power see p.45

Exponent Form	Factor Form	
10^0	$= 1$	
10^1	$= 10$	
10^2	$= 10 \times 10$	
10^3	$= 10 \times 10 \times 10$	*(Thousands: 3 zeros.)*
10^4	$= 10 \times 10 \times 10 \times 10$	
10^5	$= 10 \times 10 \times 10 \times 10 \times 10$	
10^6	$= 10 \times 10 \times 10 \times 10 \times 10 \times 10$	*(Millions: 6 zeros)*
10^7	$= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	
10^8	$= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	
10^9	$= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	*(Billions: 9 zeros)*
10^{10}	$= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	
10^{11}	$= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	

Note: The exponent number = the number of factors.

Multiplication & Exponents

Exponents are the **shorthand way** of writing multiplication when **one factor is used a number of times.** For example, instead of writing 8 x 8 x 8 x 8 x 8, we can simply write 8^5 in exponential notation. Exponents are multiplication, but with this difference:

In Multiplication:

Factors are ***different*** →

$$\underbrace{2 \times 8 \times 5}_{\text{factors}} = \underset{\underset{\textit{product}}{\uparrow}}{80}$$

In Exponents:

Factors are ***identical*** →

$$\underbrace{6 \times 6 \times 6}_{\text{factors}} = \underset{\underset{\textit{power}}{\uparrow}}{6^3} = 216$$

Connection: Exponents are used widely in math because it is a convenient way of writing large numbers.

Exponents & Powers Of 10

Powers of 10 are exponents. The difference between the two is:

In Exponents:

$9 \times 9 \times 9 \times 9 = 9^4$

$y \cdot y \cdot y \cdot y \cdot y \cdot y = y^6$

In exponents, the "***base***" can be ***any number*** (excluding zero) or ***variables.***

Variables are letters, such as a, b, c, x, y, z, etc., that stand for numbers.
A "centered dot" or parentheses (y)(y) can be used to indicate multiplication. (See p.113)

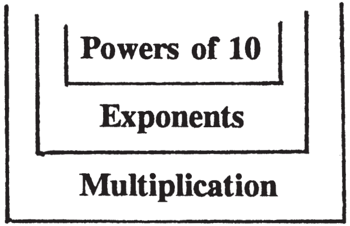
In Powers of 10:

$10 \times 10 \times 10 = 10^3$

In powers of 10, the "***base***" is ***always 10.***

Connection: The powers of 10 in exponents are used in writing scientific notation for very large and very small numbers. **(See p.74)**

Relating Multiplication, Exponents, & Powers Of 10



Points To Remember:

Multiplication, exponents, and powers of 10 are related as you see in the diagram on the left. We may consider that:

* The power of 10 is **a special case** of the exponent.

* And the exponent is **a special case** of multiplication.

Connections:

Exponents are very useful. For example, we can write:
- **place value** in exponents (powers of 10). **(See pp.36, 72)**
- numbers in **expanded form** with exponents (powers of 10). **(See p.25)**
- **prime factorization** with exponents. **(See p.319)**
- very large/small numbers in **scientific notation** (powers of 10). **(See p.74)**

And exponents are used extensively in algebra.

Exponents - Raising A Number To Powers (see p.38)

When a number is multiplied by itself once or more times, the number is raised to a power. Let's take the number 5 as an example:

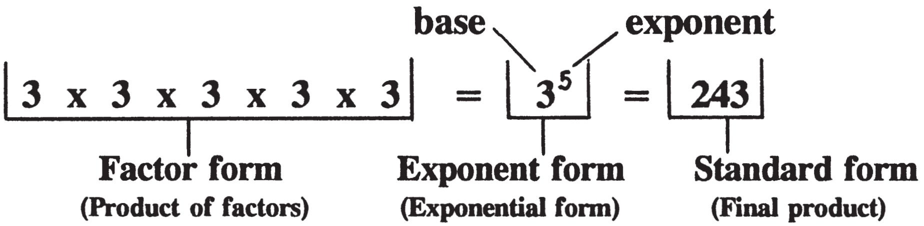
5 multiplied by itself:	5 is raised to the:	The exponents is read:
once (5 x 5)	2nd power (5^2)	"five to the **second power**" or "five **squared**"
twice (5 x 5 x 5)	3rd power (5^3)	"five to the **third power**" or "five **cubed**"
three times (5 x 5 x 5 x 5)	4th power (5^4)	"five to the **fourth power**" ...

Note: Only the "second" and the "three" powers have different names.
"Square numbers" are **two-dimensional** numbers.
"Cubic numbers" are **three-dimensional** numbers.

Reading and Writing Exponents

Base - The number (3) which is used as factors repeatedly is called base.

Exponent - The raised small number is called exponent. It tells how many times the base (3) is used as factors.



$$3 \text{ x } 3 \text{ x } 3 \text{ x } 3 \text{ x } 3 = 3^5 = 243$$

Reading Exponents - It is important that you are able to read exponential notation correctly. For example, to read 3^5

- First, read the base (3): **"three"**
- Then, add the words "to the": **"three to the"**
- Finally, read the exponent (5) as an ordinal number with the word "power": **"three to the fifth power."** 3^5

Ordinal numbers are numbers that show order or position: 1st, 2nd, 3rd,... (See p.11)

Example: Write 7 x 7 x 7 x 7 x 7 in exponent form.

Step 2. Count the **number of factors** (5).

$$\overset{\textcircled{1}}{7} \text{ x } \overset{\textcircled{2}}{7} \text{ x } \overset{\textcircled{3}}{7} \text{ x } \overset{\textcircled{4}}{7} \text{ x } \overset{\textcircled{5}}{7} = 7^5$$

Step 3. Write the **exponent: 5.**

The number of times the base has been used as a factor.

Step 1. Write the **base: 7.**

The number (7) used as a factor repeatedly.

Example: Write 3^4 (a) in factor form, (b) in standard form.

The exponent "4" means that there are 4 factors.

(a) $3^4 = \overset{\textcircled{1}}{3} \text{ x } \overset{\textcircled{2}}{3} \text{ x } \overset{\textcircled{3}}{3} \text{ x } \overset{\textcircled{4}}{3}$

(b) $3^4 = 81$ (final product)

Remember: 3^4 does not mean 3 x 4.

Working With Powers Of 10 (Read "Rule for Multiplying by 10, etc.," p.178)

If you want to get the final product of 5^6 or 2^9 , you have to multiply out the long way, two factors at a time unless you use a calculator. But to find the final product of powers of 10 can be done **with no work.** Here is the shortcut.

Powers of 10	**Number**	**Shortcut**
10^8	100,000,000	Write 1, followed by **exactly** the same number of zeros as the exponent number.
10^7	10,000,000	
10^6	1,000,000	
10^5	100,000	

Reason behind the shortcut:

$10^3 = 10 \times 10 \times 10$

The exponent (3) indicates the number of times 10 multiplied by itself or the number of factors 10.

$10 \times 10 \times 10 = \mathbf{1{,}000}$

The rule is to add one zero to the number each time it is multiplied by 10 (or add one zero for each factor 10).

The shortcut utilized the rule for multiplying by a power of 10.

The shortcut also works when writing a number as a power of 10 in exponent (or exponential form). It is just the other way around.

Number	**Powers of 10**	**Shortcut**
100,000	10^5	Write 10 (the base), then count **the number of zeros, NOT the number of digits.** Write that total as the exponent number.
10,000	10^4	
1,000	10^3	
100	10^2	

First Power & Zero Power - If the numbers above are continued in decreasing order, we would notice the following pattern:

Number	**Exponents**
10,000	10^4
1,000	10^3
100	10^2
10	10^1
1	10^0

Since each successive number decreases by a factor of 10, we can conclude that:

10^1 - **10 to the 1st power is 10.**
10^0 - **10 to the zero power is 1.**

Remember: Any non-zero number to the 1st power is the number Any non-zero number to the zero power is 1.

Summary
(Whole Numbers)

* Our number system is called the decimal system or the base-10 system because each place (digit) is 10 times larger than the place (digit) to its right.
* The decimal system implies a place value system which means the value of a digit depends on its place in the number.
* To make it easy to read, the whole numbers are grouped into periods of three digits. Any whole number can be written in standard numeral, in word, or in expanded form.
* To compare and order the numbers, line up the numbers according to their place values, then compare the digits of same value beginning from the left. The larger the number, the greater the value.
* Rounding gives an approximation when the exact answer is not required. The rounded-up number is always larger than the actual number
* Since our number system is a decimal system, the value of each place can be expressed by the powers of 10 (the multiplication of 10s) The powers of 10 are very easy to work with.
* Exponents are the shorthand way of writing numbers when one factor is used repeatedly

Part I. Numbers & Concepts

C. Decimals

Table of Contents

Decimal Point - The Dividing Point (Review first "Decimal System" p.34)

Our decimal system works also to the right of ones digit. We use **the decimal point** *to divide* **the whole numbers part from the decimals** as seen below:

Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths
100	10	1	•	0.1	0.01	0.001

To the left are: ←—— **decimal point** ——→ *To the right* are:

- * *Whole numbers*
- * Numbers equal to or greater than 1 (≥ 1)

- * *Decimals*
- * Numbers less than 1 (< 1)

moving to the left

value increases ←——————

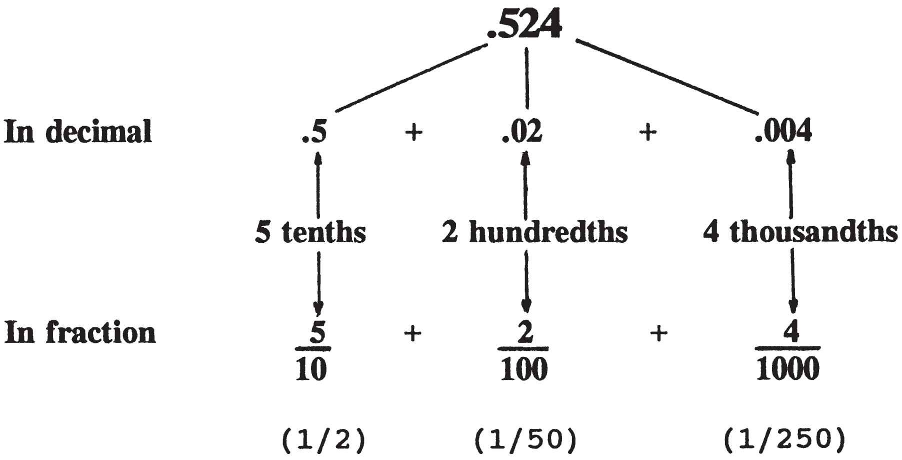
moving to the right

——————→ **value decreases**

This is how the decimal system works.

Decimals & Fractions (Read also p.225)

Decimals and fractions are **two different ways** of writing the same number. If we write 0.524 in expanded form, you will notice that each decimal digit can be expressed in both decimal and fraction.



(.524 = 524/1000)

Reducing to lowest terms changes a decimal fraction to a common fraction.

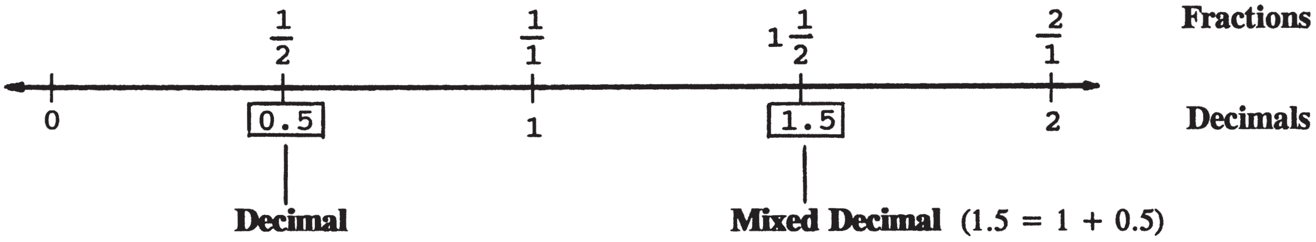
Connection: If you know that decimals can be written in fractions or the other way around, it can help to simplify the computation.

Example: .25 x 80 = 1/4 x 80 = 20

Relating Whole Numbers & Decimals/Mixed Decimals (see also p.93)

On a number line, you can find **all decimals and mixed decimals. They are numbers that lie between some pairs of whole numbers.** The difference between the two is:

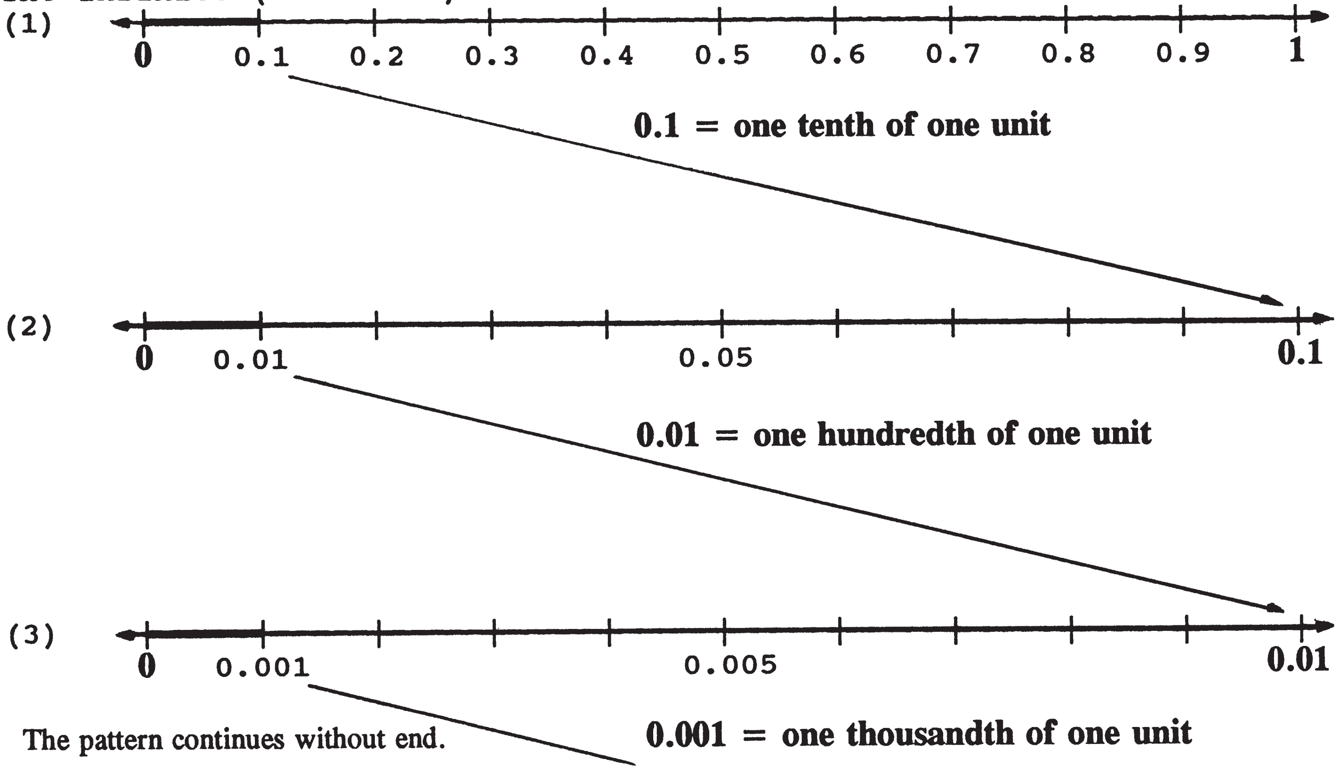
* **Decimals** are *less than 1* (< 1) - (no whole number part)
* **Mixed decimals** are *greater than 1* (> 1) - (whole number + decimal)



With the help of number lines, we can see a **major difference** between ***the set of whole numbers*** and ***the set of decimals/fractions***:

* Between any pair of whole numbers, 0 & 1, 1 & 2, 5 & 6, 11 & 12, etc., **it is impossible to find another whole number.** However,
* Between any pair of whole numbers or any two decimals (fractions), there are an **infinite (unlimited) number** of decimals/fractions. On the next page you will see unlimited decimals just between 0 & 0.1:

The infinite (unlimited) decimals between 0 & 0.1:



0.1 = one tenth of one unit

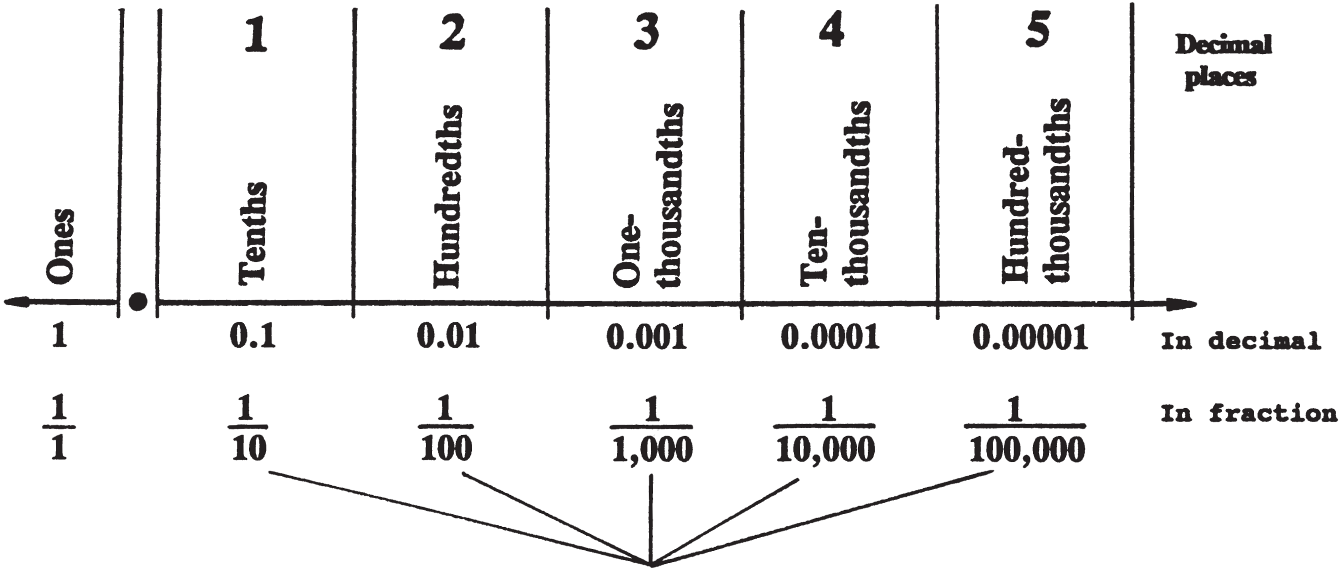
0.01 = one hundredth of one unit

0.001 = one thousandth of one unit

The pattern continues without end.

Chart: Place Value Of Decimals (Review first p.50)

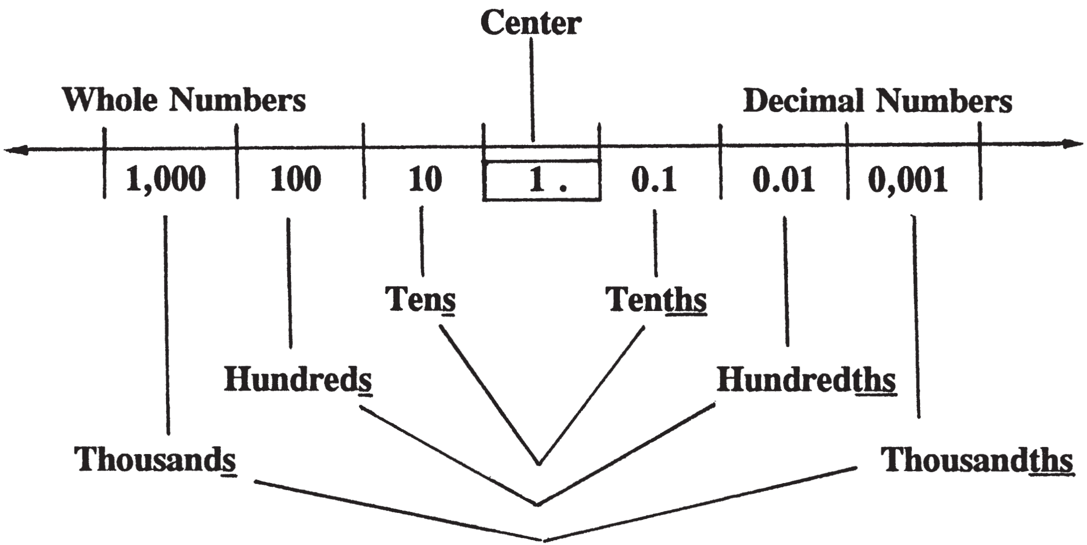
The chart below gives the place value of the digits *to the right* **of the decimal point.** On a number line, you will find these decimal points lie between 0 and 1 as illustrated on the previous page.



The denominators are powers of 10

The following suggestion may help you to visualize the place value of whole numbers and decimal numbers.

Consider 1 and decimal point (.) as the center.



Do you see the relation?

Number of Decimal Places Or Decimal Digits (Review first p.54)

You need to have the concept of "decimal places" and their names before you can translate correctly a decimal in words (such as "three hundredths") into a decimal in standard numeral or vice versa. **The number of decimal places** means **the number of digits** a number has **after the decimal point.** Compare with "Digits" on page 225.

Place after decimal point	Word name of the place	Number of "decimal places"
1st	Tenths	1 place
2nd	Hundredths	2 places
3rd	Thousandths	3 places
4th	Ten thousandths	4 places
5th	Hundred thousandths	5 places

Memorize the first four place names and the number of decimal places!

Points To Remember:

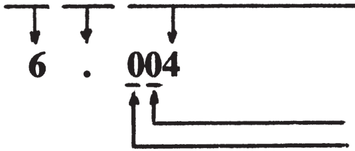
* The "**th**" at the end of word name indicates a decimal number.
* The "**and**" and "**th**" at the end of word name indicate a mixed number.

Translating Words into Decimals and Fractions (Review first p.54)

Decide first ***how many decimal places*** the number has as indicated by the ***word name of the place,*** then you know whether or not zero(s) are needed to hold the vacant place(s). Study carefully the following examples.

1. Write "six and four thousandths" in decimal and fraction.

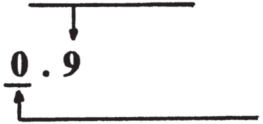
six and four *thousandths* = 6.004 or 6 4/1000



The word name "**thousandths**" indicates the number has **3 decimal places.** Since 4 is in "thousandths" place, "**0s**" are needed ***to hold*** the ***"tenths"*** & ***"hundredths"*** places.

2. Write "nine tenths" in standard form or standard numeral.

nine *tenths* = 0.9 or 9/10



Write a zero in "**ones**" place for numbers **less than 1**, because it is easy to mistake **.9** (nine tenths) for **9** (nine).

Reading & Writing Decimals in Words - Use "Place Value Charts" for help.

1. Reading **Decimals** - numbers between 0 and 1.

(a) **0.409**

four hundred nine thousandths

Read the decimal as you would with whole number Then, read the place-value name of the last digit (3rd place).

2. Reading **Mixed Decimals** - numbers greater than 1.

(b) **2.016**

two and sixteen thousandths

Read the whole number part first, then "and" for decimal point, then the fractional part (decimal).

3. Reading **Equivalent Decimals** - decimals which have same value.

(c) **0.7 = 0.70**

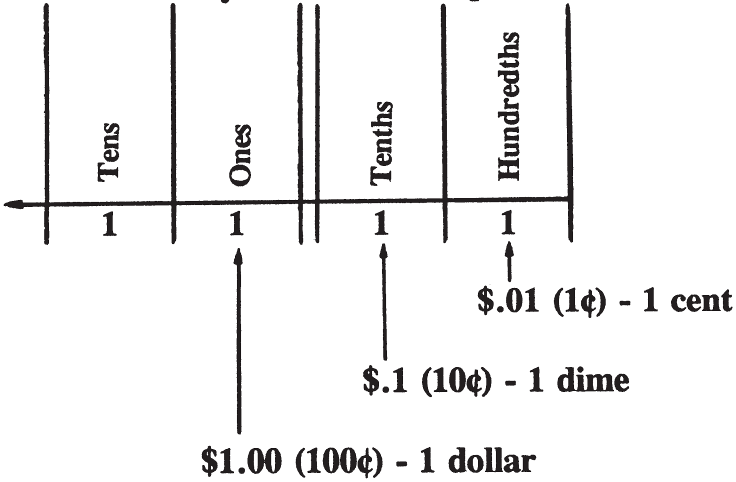
seven ***tenths*** **seventy** ***hundredths***

Place zero(s) after a decimal does not affect its value, but it does affect the way the decimal is read.

Connection: We use "equivalent decimals" in **comparing & ordering** decimals.

Decimals and Money (Read also p.227)

Our money system works in the same way as our decimal system except it involves **only two** decimal places - the **"tenths" (dimes) & "hundredths" (cents).**



1 dollar = 2 half dollars
= 4 quarters
= 10 dimes
= 20 nickels
= 100 pennies

1 quarter = 2 dimes & 1 nickel
= 5 nickels
= 25 pennies

1 dime = 2 nickels
= 10 pennies

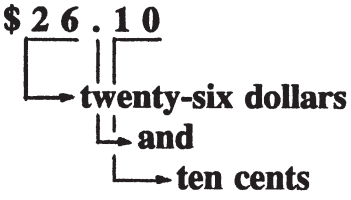
1 nickel = 5 pennies

There are two ways to write money:
- using dollar sign ($) and decimal point (.) → **$.00**
- using the cent sign (¢) for the amount **less than one dollar.**

Reading and Writing Money (Review first p.59)

Read and write money the way you would with whole numbers and decimals; however, add the word "dollar" after the whole numbers and "cent" after the decimal numbers.

(a) **$26.10**



Reading money:
1st, read the whole number part **ending with "dollar"**. Then, read **the decimal point with "and"**. Finally, read the **decimal part** followed by the word **"cent"**.
Read: "twenty-six dollars and ten cents."

(b) **"five dollars and fifty-nine cents"**

$5.59

Writing money:
1st, write the dollar sign **"$"** followed by the whole number. Then write the decimal point "." for "and", followed by **2 decimal numbers** (2 places).

(c) **"forty-seven cents"**

$.47 or 47¢

Write the dollar sign with a dot **"$."** followed by the decimal numbers. Or write the decimal numbers with **cent sign.**

Rounding Decimals & Money (Review first "Rounding Off," p.30)

Rounding decimals and rounding money follow the same rule as rounding whole numbers. The digit to the right of the place you are rounding determine whether to round up or to round down. The following examples show the one difference between rounding money and rounding decimal.

Example 1: Rounding $19.58 to the nearest dime.

$$\$19.\overset{\downarrow}{5}8 \longrightarrow \$19.60$$

With money, we always keep **two** decimal places - the "***dime***" (tenths) and the "***cent***" (hundredths). **(See also p.59)**

Example 2: Rounding 19.58 to the nearest tenths.

$$19.\overset{\downarrow}{5}8 \longrightarrow 19.6$$

With decimal, we drop zero(s) after the decimal number. **Dropping** the "**0s**" after a decimal ***does not*** change its value: 19.60 = 19.6. They are equivalent decimals. **(See also p.67)**

Writing Decimals & Money In Expanded Form (Review first, p.24)

The process of writing decimals and money in expanded form is the same as writing whole numbers in expanded form. **The key is to recognize the place value of each digit.** The following examples show how to write (a) 32.759 and (b) $164.36 in expanded form:

(a) **32.759**

= 3(10) + 2(1) + 7(0.1) + 5(0.01) + 9(0.001)

digit x (place value)

= **30 + 2 + 0.7 + 0.05 + 0.009** ← **32.759 in expanded form**

(b) **$164.36**

= $1(100) + $6(10) + $4(1) + $3(0.1) + $6(0.01)

digit x (place value)

= **$100.00 + $60.00 + $4.00 + $.30 + $.06** ← **$164.36 in expanded form**

To avoid errors, always write money with two decimal places: $100.00 instead of $100, and $.30 instead of $.3

Writing Decimals (Expanded Form) in Standard Form **(Review first p.26)**

Again, the method is the same for decimals as it is for the whole numbers. If you know the place value well, you can write the number, whole or decimal, in standard form with no work. However, if you need to add the numbers, it is important that you **line up the numbers in column correctly.** For example, write (a) 4,000 + 70 + 3 and (b) 10 + 5 + 0.1 + 0.006 in standard form.

(a)

$$\begin{array}{r} 4{,}000 \\ 70 \\ +\quad\quad 3 \\ \hline 4{,}073 \end{array}$$

Standard numeral

For whole numbers, use ones digit as a lead to line up the numbers in column according to their place values.

Add 0s to the empty places.

(b)

$$\begin{array}{r} 10 \\ 5 \\ 0.1 \\ +\ 0.006 \\ \hline \end{array} \qquad \begin{array}{r} 10.000 \\ 5.000 \\ 0.100 \\ +\ 0.006 \\ \hline 15.106 \end{array}$$

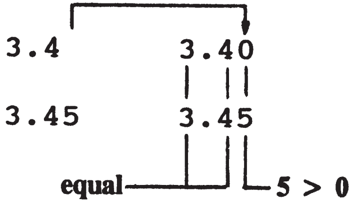
Standard numeral

For decimal numbers, line up the decimal point first. Then add "0s" to the empty places in the columns to make them even in order to avoid errors in computation. **(See also p.67)**

Comparing and Ordering Decimals **(Review first "Symbols for Comparing" p.28)**

1. **Compare** 3.4 and 3.45. Use one of the following methods:

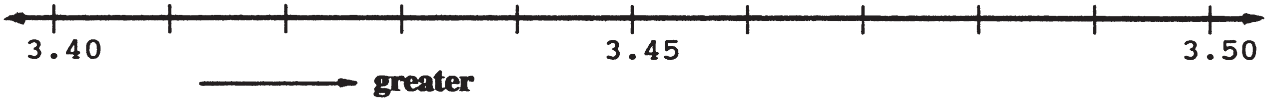
a) **Compare digit-by-digit.**
Write the numbers vertically with **the decimal points lined up** which means to line up the numbers according to their place values. **Add zeros if needed** so all the decimals would have the same numbers of digits. Then compare decimals as you would with whole numbers **(See p.29)**



Starting from left, compare digits of the same value until you find a pair of digits that are **not equal** - "hundredths" digit.

Since 5 > 0, 3.45 > 3.4 or 3.4 < 3.45

b) **Compare on a number line.**



General rule: On a number line like this, the number *on the right* is *greater* than the number on the left. **So, 3.45 > 3.4** or **3.4 < 3.45**

c) **Compare by using expanded form.**
Write each number as the sum of its digits and compare.

$3.4 = 3.00 + 0.4 + \mathbf{0.00}$
$3.45 = 3.00 + 0.4 + \mathbf{0.05}$

The first place they are not equal is hundredths. **Since $0.05 > 0$, $3.45 > 3.4$**

> **Note: Be careful in interpreting the result of comparison. For example, if the numbers represent the timing of a race contest, the lesser number won the race. If they represent the distance of frog jumps, the greater number won the contest.**

2. **Ordering from least to greatest:** 3; 0.967; 1.7; 0.96.
Write the numbers vertically with the decimal points lined up. Add zeros if needed so all the numbers have the same number of digits. Compare first the whole number part and then the decimal part, **two numbers at a time.**

3 → 3.000
0.967 → 0.967
1.7 → 1.700
0.96 → 0.960

* Compare ones place: $3 > 1$. So, $3 > 1.7$
* Compare two decimals. **The first place** that are different is the thousandths. **Since $0.007 > 0$, therefore $0.967 > 0.96$.**

Order from least to greatest: $0.96 < 0.967 < 1.7 < 3$

> **Note: $3 = 3.000$; $1.7 = 1.700$; $0.96 = 0.960$ They are equivalent decimals.**

(See also p.67)

Adding Zeros Before Whole Numbers And Decimals

Whole Numbers

3
03
003
0003
00003

3 remains in "ones" place.

* Adding zeros (0s) **before** a whole number ***does not*** change the value of the number:
0003 = 003 = 03 = 3

* Therefore, we **drop 0s** that come before a whole number in division when the divisor is larger than the dividend.

Decimals (See also p.69)

.3 —— 3 tenths
.03 —— 3 hundredths
.003 —— 3 thousandths
.0003 —— 3 ten-thousandths
.00003 —— 3 hundred-thousandths

* Adding zeros **immediately after** the decimal point and **before** a decimal digit ***changes*** the value of the decimal.

* **Each zero** you add, ***reduces*** the value of the number **to 1/10:**
  - .03 is only 1/10 of .3
  - .003 is only 1/10 of 03

Suggestion: Study this page and next page side by side carefully.

Adding Zeros After Whole Numbers And Decimals

Whole Numbers (See also p.178)

3 — 3
30 — 3 tens
300 — 3 hundreds
3000 — 3 thousands
30000 — 3 ten-thousands

* Adding zeros **after** a whole number ***changes*** the value of the number.

* **Each zero** you add, ***increases*** the value of the number **10 times:**
  - 30 is 10 times larger than 3
  - 300 is 10 times larger than 30.
  - 300 is 100 times larger than 3

Decimals

.3
.30
.300
.3000
.30000

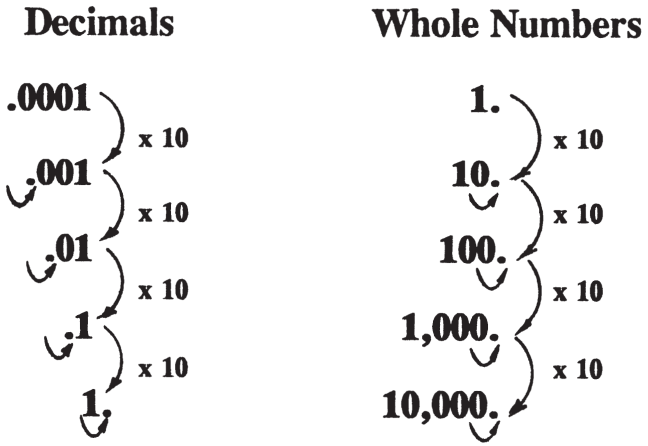
3 remains in the "tenths" place.

* Adding/dropping "0s" **after** a decimal number ***does not*** change the value of the decimal:
  - .3 = .30 = .300 = .3000
  - .3000 = .300 = .30 = .3

* Therefore, we add 0s in adding/subtracting/comparing decimals to avoid error.

Note: Do you see that the effects on whole numbers and decimals are just opposite?

Multiplying Numbers by 10, 100, etc. & Decimal Places (See also p.228)



To multiply a number by 10, the decimal point is moved 1 place to the right and the value of the number increases 10 times.

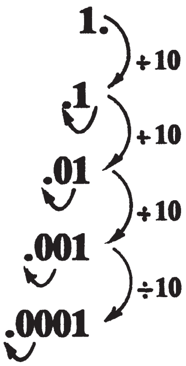
For whole numbers, the decimal point is at the right of ones digit which is usually omitted.

General Rule:

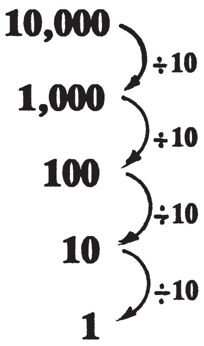
To multiply a decimal:	by **10**	by **100**	by **1000** ...
Move the decimal point **to the right:**	**1** place	**2** places	**3** places ...

Dividing Numbers by 10, 100, etc. & Decimal Places (see also p.229)

Decimals



Whole Numbers



To divide a number by 10 equals to multiplying by its reciprocal 1/10.

To divide a number by 10, the decimal point is moved 1 place to the left and the value of the number decreases by 1/10.

Use zeros to hold places if necessary.

General Rule:

To divide a decimal:	by **10**	by **100**	by **1000** ...
Move the decimal point **to the left:**	**1** place	**2** places	**3** places ....

Powers of 10 with Negative Exponents (See also "Place Value in Exponents" p.72)

If the numbers found on page 45 **(First Power & Zero Power)** is continued in decreasing order, we will see the following pattern:

The Number		**Exponent**
10		10^1
	x 1/10	
1		10^0
	x 1/10	
$\frac{1}{10}$		10^{-1}
	x 1/10	
$\frac{1}{100}$		10^{-2}
	x 1/10	
$\frac{1}{1,000}$		10^{-3}
	x 1/10	
$\frac{1}{10,000}$		10^{-4}

Do you observe that when we multiply each successive number by 1/10, which equals to divide by 10, each number is **decreased by a factor of 10.** And it leads to ***negative exponents.***

Powers of 10 with negative exponents can be written as positive exponents *in fraction with 1 as the numerator*:

$$10^{-3} = \frac{1}{10^3}, \quad 10^{-7} = \frac{1}{10^7}$$

Difference Between Positive & Negative Exponents **(Read also p.75)**

It will help you to see the difference between numbers with positive and negative exponents if we write them in fraction form.

Remember: The exponent indicates the number of times the base is used as a factor.

(a) $10^3 = \frac{10 \times 10 \times 10}{1} = 1,000$

With **positive** exponent (3), the factors are **in the numerator.**

$10^{-3} = \frac{1}{10 \times 10 \times 10} = \frac{1}{1,000}$

With **negative** exponent (-3), the factors are **in the denominator.**

This is also true of any base other than the base of 10 (powers of 10).

(b) $3^4 = \frac{3 \times 3 \times 3 \times 3}{1} = 81$

With **positive** exponent (4), the factors are **in the numerator.**

$3^{-4} = \frac{1}{3 \times 3 \times 3 \times 3} = \frac{1}{81}$

With **negative** exponent (-4), the factors are **in the denominator.**

Note: 10^{-3} (1/1000) and 10^3 (1000) are **reciprocal** of each other. **(See p.326)**

Chart: Place Value In Exponents

The value of each place can be expressed in different forms:

Whole numbers Positive exponents				Decimal numbers Negative exponents			
1	1	1	●	1	1	1	Place value in:
100	10	1		0.1	0.01	0.001	Standard form
$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$	Fraction form
$\frac{10\cdot10}{1}$	$\frac{10}{1}$	$\frac{1}{1}$		$\frac{1}{10}$	$\frac{1}{10\cdot10}$	$\frac{1}{10\cdot10\cdot10}$	
$\frac{10^2}{1}$	$\frac{10^1}{1}$	$\frac{10^0}{1}$		$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	Exponent form Positive & Negative
10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}	

The chart below shows the place value of the first five decimal places, the places *to the right* of the decimal point.

In Word	In Decimal Number	In Decimal Fraction	In Exponent
Ones	1	$\frac{1}{1}$	10^{0}
Tenths	0.1	$\frac{1}{10}$	10^{-1}
Hundredths	0.01	$\frac{1}{100}$	10^{-2}
Thousandths	0.001	$\frac{1}{1,000}$	10^{-3}
Ten-thousandths	0.0001	$\frac{1}{10,000}$	10^{-4}
Hundred-thousandths	0.00001	$\frac{1}{100,000}$	10^{-5}

↓ The value of the decimal decreases.

↓ The denominator becomes bigger.

Scientific Notation (Review first "Exponents & Powers of 10," p.39)

Scientific Notation is a convenient way of writing large and small numbers **as a product of two factors** in the following way:

(The first factor must be a number greater than or equal to 1 but less than 10.) **X** **(The second factor is a power of 10 written in exponent form.)**

The **second** factor can be a power of 10 with either:

1. a **positive** exponent - for numbers greater than 10, or
2. a **negative** exponent - for numbers less than 1.

(The above statements are true if the exponents are integers.)

The **first** factor can be either:

1. a **whole number** such as 1, 2, 3, ..., 9, ***but not 10.***
   (Note: 10 can be written in scientific notation by itself as 1×10^1.) Or
2. a **mixed decimal** (***not a decimal***) greater than 1 but less than 10.

Example: Are 10.35×10^3 and 0.95×10^2 written in scientific notation?
Ask yourself: "Are the first factors greater than 1 and less than 10?"
Answer: "No". Neither one is written in scientific notation.

Writing Scientific Notation with Positive & Negative Exponents (Review p.71)

with Positive Exponents

The number given must be **greater than 10.**

Step 1.

Divide the number by a power of 10, reducing it to a **smaller** number, a number between 1 and 10. This is the first factor.

Step 2 is the inverse operation of Step 1.

Step 2.

Multiply the reduced number by the power of 10, by which we divided the number in step 1. This is the second factor.

See examples on pp. 76-77

with Negative Exponents

The number given must be **less than 1.**

Step 1.

Multiply the number by a power of 10, making it a **larger** number, a number between 1 and 10. This is the first factor.

Step 2 is the inverse operation of Step 1.

Step 2.

Divide the number by the power 10, by which we multiplied the number in step 1. This is the second factor.

(To divide by a power of 10 = to multiply by the power of 10 with a negative exponent.)

See an example on p.80

Writing in Scientific Notation - Example 1 shows the process by giving a step-by-step analysis. Examples 2 & 3 use the shortcut.

Example 1: Write 275 (**whole number**) in scientific notation.

$$275. \xrightarrow[\div 10]{} 27.5 \xrightarrow[\div 10]{} 2.75$$

To divide a number by 10, move the decimal point 1 place to the left. Moving 2 places to the left when dividing it by 100 or 10^2.

$$275 \div 100 = 2.75$$

$$2.75 \times 100 = 275$$

$$2.75 \times 10^2$$

275 in scientific notation

Step 1.

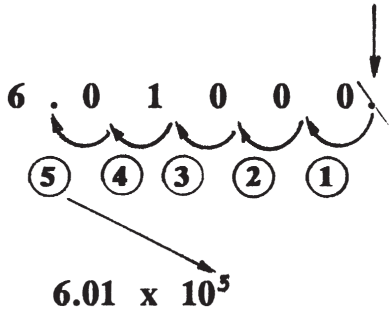
Divide 275 by 10 successively (powers of 10), until 275 is reduced to 2.75, a number between 1 and 10. Write 2.75 as the first factor.

Step 2 is the **inverse operations** of **Step 1.**

Step 2.

Multiply 2.75 by the powers of 10 (10^2) by which we divided the number 275 in step 1. This is the second factor.

Example 2: Write 601,000 (**whole number**) in scientific notation.

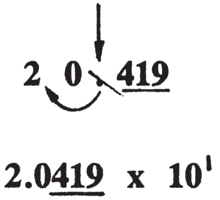


Step 1. Locate the decimal point. It is at the end of "ones" digit.

Step 2. Place the decimal point between 6 and 0 and count how many places the decimal point was moved to the left. The number of places moved is the exponent number.

Step 3. Write 6.01 as the first factor and 10^5 as the second factor. Since 6.01000 = 6.01, drops the 3 zeros.

Example 3: Write 20.419 (**a mixed decimal**) in scientific notation.



- * Locate the decimal point and move it to the place where the number will be reduced to just between 1 and 10.
- * In scientific notation, the 1st factor **must include the decimal part - 419.**
- * $10^1 = 10$, the exponent may be omitted.

Changing Scientific Notation To Standard Form (Review first "Powers of 10" p.37)

Example: Write 4.27×10^6 in standard numeral.

$$4.27 \times 10^6$$

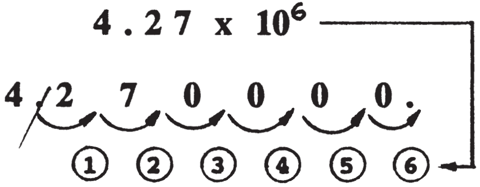
$$\begin{array}{r} 4.27 \\ \times\ 1{,}000{,}000 \\ \hline 4{,}270{,}000 \end{array}$$

Method 1: Long Way

Multiply 4.27 by 1,000,000 like an arithmetic problem.

← The standard numeral is 4,270,000.

The exponent indicates the number of places the decimal point is to be moved to the right.



Method 2: Shortcut

1st, look at the exponent (6). Then moves the decimal point 6 places to the right, add "0" to fill the places.

4,270,000

← The standard numeral is 4,270,000.

Multiplying Numbers by Powers of 10 with Negative Exponents (Review p.71)

Study the following example carefully until you understand what it means to multiply a number by a power of 10 with a negative exponent.

$$5 \times 10^{-2} = 5 \times \frac{1}{10^2} = 5 \times \frac{1}{100} = \frac{5}{100} = 5 \div 100 = 0.05$$

To multiply by 10^{-2} equals to divide by 10^2

Points To Remember:

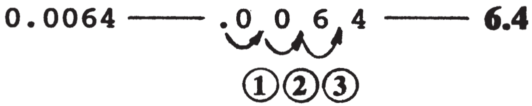
* When writing in scientific notation:
- if the given number is **greater than 10**, the power of 10 has **a positive exponent.**
- if the given number is **less than 1**, the power of 10 has **a negative exponent.**

* If the exponents are **integers** (whole numbers), multiplying by:
- a power of 10 with **positive** exponent, makes the number **larger**.
- a power of 10 with **negative** exponent, makes the number **smaller**.

The above statements are also true of any base(nonzero numbers) other than the base 10, if the exponents are integers.

Writing in Scientific Notation with Negative Exponents

The following example shows you how to write 0.0064, a number less than one, in scientific notation.



Moving the decimal point 3 places to the right = multiplying the number by 1000.

$$0.0064 \times 10^{3} = 6.4$$

$$6.4 \times 10^{-3} = 0.0064$$

$$6.4 \times 10^{-3}$$

0.0064 in scientific notation

Step 1. Write the first factor.
Multiplying by 1000 (= 10^{3}) changes 0.0064 to 6.4 -- a larger number, a number greater than 1 and less than 10. Write 6.4 as the first factor.

Step 2 is an inverse operation of Step 1. To divide by 1000 equals to multiply by 10^{-3} (1/1000).

Step 2. Write the second factor.
Multiply 6.4 by 10^{-3} . Multiplying by 10 with a negative exponent (-3) reduces 6.4 to 0.0064, the original number.

Changing Scientific Notation with Negative Exponents to Standard Form (p.37)

The following shows how to write 9.1×10^{-5} in standard numeral.

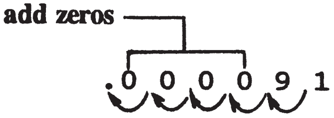
Remember: To multiply by 10^{-5} is to divide by 10^{5} (= 100,000).

$$\begin{array}{r} .000091 \\ 100000\overline{)9.100000} \\ -\underline{9\ 00000} \\ 100000 \\ -\underline{100000} \\ 0 \end{array}$$

Method 1. Long Way

Divide 9.1 by 100,000 (10^{-5}) as you would divide a decimal.
The standard numeral: **.000091**

9.1×10^{-5}



0.000091

9.1×10^{-5} in standard numeral

Method 2. Shortcut

First, look at the exponent number. Since negative exponent (-5) means division, move the decimal point 5 places to the left. Fill the places with zeros (0s).

To move the decimal point 5 places to the left is the same as to divide the number by 100,000 (or 10^{-5}).

Summary
(Decimals)

- On a place value chart, the decimal point is the dividing point - to its left are whole numbers and to its right are decimals. The decimal point of a whole number is at the right of the ones digit which is usually omitted.
- Decimals are not grouped into periods of three digits like whole numbers. To read decimals, one must know the names of the decimal places.
- Our money system is similar to the decimal system except it keeps only two decimal places - the tenths (dime) & the hundredths (cent)
- Adding zeros before the whole numbers or after the decimal numbers does not affect the value of the numbers. But adding zeros after the whole numbers, or between the decimal point and a decimal digit changes the value of the number
- To multiply decimals by powers of 10, the decimal points is moved to the right and the number becomes larger; to divide decimals by powers of 10, the decimal point is moved to the left and the number becomes smaller
- Scientific notation is a convenient way of writing very large numbers and very small numbers. If a given number is greater than 10, its power of 10 has a positive exponent; if a given number is less than 1, its power of 10 has a negative exponent.

Part I. Numbers & Concepts

D. Fractions

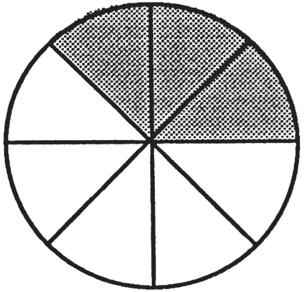
Table of Contents

Meanings Of Fractions (a/b) (Read first p.8)

Keep in mind that numbers written in fraction form (a/b) could be used in one of the following sense:

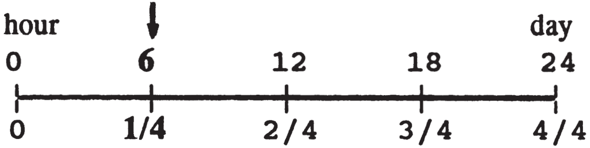
1. Fractions show **"relationship"** - **"a part of the whole."**

   Example: "A pizza was cut into 8 ***equal*** pieces, and Joe ate 3 pieces."



3 Joe ate 3 pieces
— out of
8 8 EQUAL pieces, the whole

Example: "Sue spent 1/4 of a day in school."



A day was divided into 4 EQUAL length of time. Sue spent one out of 4 EQUAL length of time in school.

In daily life, it is common to use fractions in this sense in dealing with **measurements.** The emphasis is placed on the word **"equal"** - the whole being divided into equal parts.

2. Fractions are used in **"comparing"** two numbers or quantities known as **"ratios."** (See p.372)

Example: "There are 5 girls and 7 boys in the math club. The ratio of girls to boys is:

$$\frac{5}{7}$$

The above fraction, 5/7, can also be expressed as:

$$5 : 7 \quad \text{or} \quad 5 \text{ to } 7$$

Note: Fractions representing ratios are not always treated in the same way as other fractions in computation. (See "Ratios" p.372)

3. Fraction means **"division"** - one of the three ways of writing division (See p.198)

Example: "4 divided by 9" can be written as:

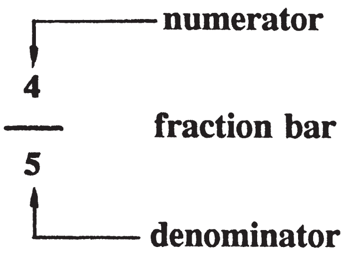
$$\frac{4}{9} \quad \text{or} \quad 4 \div 9 \quad \text{or} \quad 9\overline{)4}$$

The quotient of 4 divided by 9 is a decimal. Instead of writing a decimal, we write as a fraction of two whole numbers and call it an **"indicated quotient"** - meaning **division as yet to be carried out.**

Connection: In algebra division is written only in fraction form.

Terms Of A Fraction & Unit Fractions

A fraction, unlike a whole number, is made up of **two numbers** divided by a horizontal line called fraction bar. The top number is called the numerator and the bottom number the denominator. They are the **two terms** of a fraction. That's why we say the fraction is in **lowest terms** when it is in **simplest form**.(p.293)



The numerator tells the number of the equal parts (a part of whole) being involved.

Fraction bar indicates "division" (÷)

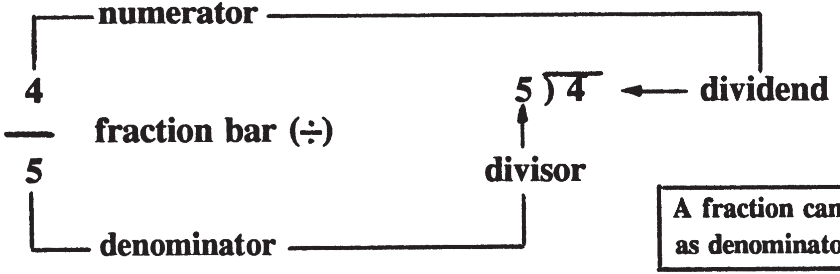
The denominator tells the number of equal parts the whole is divided into.

Unit Fraction: A unit fraction is a fraction with **a numerator of 1:**

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ Unit fractions are proper fractions.

Fractions & Division

Keep in mind that ***fractions means division.*** Fraction form is one of the three ways of writing a division (See p.198) However, in arithmetic computation (not algebra), division is done in "$\overline{)\quad}$" form. The following shows you how to change fractions to division:



A fraction can have any whole number as denominator except zero.

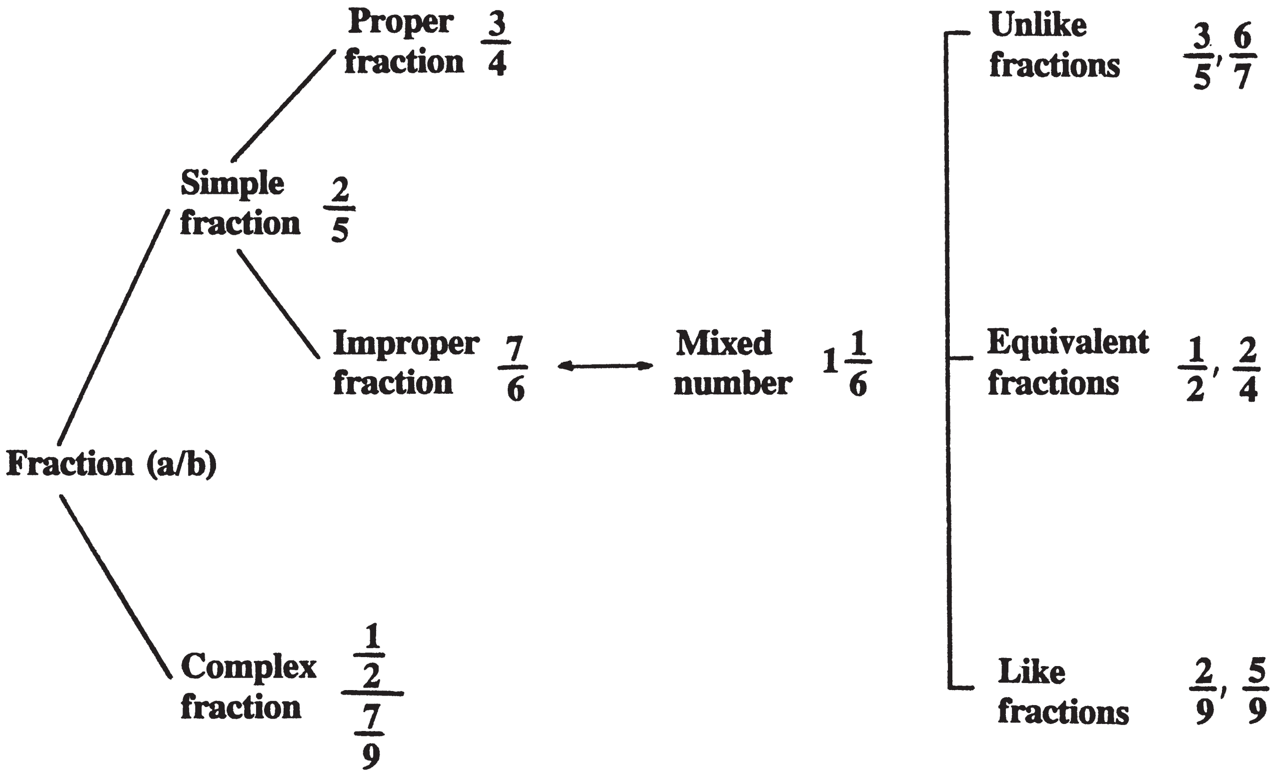
Reading Fractions: Using 4/5 as an example:

1. If it implies **relationship,** read as: **"four fifths."** (concrete number)

When writing fractions in words, the first number is always the numerator

2. If it indicates **division,** read as: **"four divided by five."** (abstract number)

Kinds of Fractions



Definition: Fractions, Simple Fractions, Complex Fractions

$$\frac{a}{b}$$

Fraction - The word "fraction" comes from the Latin *"fractus"* which means *"broken."* That's why some call fractions, the broken numbers. **On a number line,** the points of fractions (*except* those equals to 1 and other **whole numbers)** lie between **some pair of whole numbers.**

$$\frac{2}{5}, \quad \frac{7}{3}$$

Simple Fraction - A simple fraction has ***whole numbers*** for **both** the numerator and the denominator, excluding "0" as the denominator.

ⓐ ⓑ ⓒ

$$\frac{\frac{1}{2}}{\frac{7}{9}}, \quad \frac{4}{\frac{1}{3}}, \quad \frac{\frac{2}{9}}{6}$$

Complex Fraction - A complex fraction has ***fractions*** for **either** the numerator **or** the denominator **or both.** They are really **division of fractions** written in **fraction form** (See p.365). Examples on the left are:

ⓐ **A fraction divided by a fraction:** 1/2 - 7/9

ⓑ **A whole number divided by a fraction:** 4 - 1/3

ⓒ **A fraction divided by a whole number:** 2/9 - 6

Definition: Proper Fractions, Improper Fractions, & Mixed Numbers

$$\frac{1}{8}, \quad \frac{1}{2}, \quad \frac{3}{4}$$

Proper Fraction - In proper fractions, **the numerator *is smaller* than the denominator.** They are decimals (less than 1) in value.

$$\frac{3}{2}, \quad \frac{7}{3}$$

Improper Fraction - In improper fractions, **the numerator is either *equal to*** or ***larger than* the denominator.** Improper fractions also include:

$$\frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = 1$$

a) **One (1)** - When the numerator ***equals*** the denominator **(See p.299)**

$$\frac{2}{1} = 2, \quad \frac{3}{1} = 3$$

b) **Whole Numbers (2, 3, 4,...)** - When the denominator is **1** **(See p.298)**

$$1\frac{1}{2}, \quad 3\frac{2}{5}$$

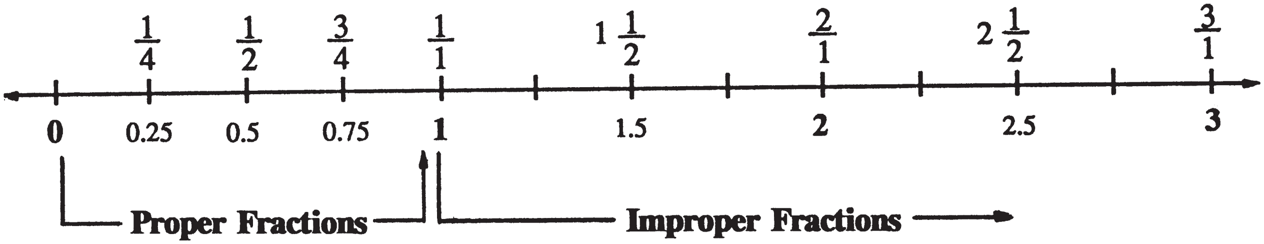
Mixed Number - A mixed number is an **improper fraction** written as **the sum of a whole number and a proper fraction with the plus "+" omitted.**

Example: (an improper fraction) 3/2 = 1 1/2 (a mixed number)

Read the mixed number 1 1/2 as **"one and one half."** **(See p.98)**

Proper Fractions & Improper Fractions on A Number Line (see also p.52)

A number line can help us to see the difference between the proper fractions and improper fractions:



Decimals & Mixed Decimals (see also p.52)

If we **change the fractions to decimals** by dividing the numerator by the denominator, we will notice that:

- * **Proper Fractions** are like **decimals** (less than 1). Their points lie between 0 and 1 on the number line as seen above.
- * **Improper fractions (Mixed Numbers)** are like **mixed decimals** (greater than 1). Their points lie between some pair of whole numbers on the number line.

Changing Improper Fractions to Mixed Numbers (Read first p.92)

Changing 21/6 to A Mixed Number:

Shortcut

$$\begin{array}{r} 3\ \ 3/6 \\ 6\,\overline{)\ 21} \\ -\,18 \\ \hline 3 \end{array}$$

Divide the numerator by the denominator and write the remainder as a fraction – remainder over divisor – and simplify:
3/6 = 1/2.   21/6 = 3 1/2.

Long Way

$$\frac{21}{6} = \frac{18+3}{6}$$

1st. Write the numerator as the sum of two numbers. The first number is the largest multiple of 6 but is smaller than 21. **(6 x 3 = 18, 18 < 21)**

$$\frac{18\ +\ 3}{6} = \frac{\mathbf{18}}{\mathbf{6}} + \frac{3}{6}$$

2nd. Write it as the sum of two like fractions. **The first fraction is an improper fraction.**
(It's the reverse process of adding like fraction.)

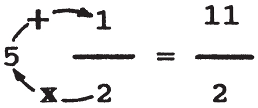
$$\frac{18}{6} + \frac{3}{6} = 3 + \frac{1}{2} = 3\,\frac{1}{2}$$

3rd. Simplify both fractions and write as a mixed number **with (+) sign omitted.**

Changing Mixed Numbers to Improper Fractions

Changing 5 1/2 to An Improper Fraction:

Shortcut



Multiply the whole number by the denominator (2) and add the product to the numerator.

Long Way

$$5\ 1/2 = 5 + \frac{1}{2}$$

1st. Write the mixed numbers as whole number + fraction.

$$= \frac{5}{1} + \frac{1}{2}$$

2nd. Write the whole number as a fraction with the denominator of 1.

$$= \frac{5 \times 2}{1 \times 2} + \frac{1}{2}$$

3rd. Change 5/1 to an equivalent fraction with 2 as the denominator.

$$= \frac{10}{2} + \frac{1}{2} = \frac{11}{2}$$

4th. Add the like fractions.

Note: The shortcut omitted step 1 & 2, and combined steps 3 and 4.

Whole Numbers Division & Mixed Numbers (See also pp.89,251)

If you write the *whole number* **divisions** in fraction form, you will notice that every one of them forms an "**improper fraction**" - the numerator larger than the denominator as the following examples show:

(a) $9 \div 3 = \frac{9}{3}$ (b) $14 \div 4 = \frac{14}{4}$ (c) $36 \div 6 = \frac{36}{6}$

When we divide 9 by 3 or 36 by 6, they divide evenly with zero remainder. But when we divide 14 by 4, we get 3 and 2 **left over** which we called ***remainder***. And the quotient of 14 divided by 4 can be written as:

a whole number	**a mixed decimals**	**a mixed number**
$\begin{array}{r} 3\ \text{r2} \\ 4\,\overline{)\,14} \\ \underline{-\,12} \\ 2 \end{array}$	$\begin{array}{r} 3.5 \\ 4\,\overline{)\,14.0} \\ \underline{-\,12} \\ 2\ 0 \\ \underline{-2\ 0} \\ 0 \end{array}$	$\begin{array}{r} 3\ 1/2 \\ 4\,\overline{)\,14} \\ \underline{-\,12} \\ 2 \end{array}$

As you see that **mixed numbers** arise often from **division of** ***whole numbers.***

Definition: Unlike Fractions, Like Fractions, Equivalent Fractions

When we deal with unlike fractions or like fractions, we are dealing with two or more fractions. The difference between the unlike and like fractions is found in their *denominators.*

$\frac{3}{5} \quad \frac{6}{7}$

Unlike Fractions - Unlike fractions have **different numbers** for the denominators.

$\frac{2}{9}, \quad \frac{5}{9},$

Like Fractions - Like fractions have the **same numbers** for the denominators.

$\frac{1}{2}, \quad \frac{2}{4}, \quad \frac{3}{6}, \ldots$

Equivalent Fractions - Equivalent fractions have the **same value** and the **same point** on a number line, but they have *different numbers* for the numerators and for the denominators. Any fraction has a very large set of equivalent fractions. Equivalent fractions are used:

a) in *changing* **unlike** fractions to **like** fractions by using LCM/LCD.

b) in *reducing* a fraction to **lowest terms** by using GCF.

$\frac{24}{32} = \frac{3}{4}$

Decimals & Decimal Fractions (Review first "Place Value of Decimals" p.54)

Decimal fractions and decimals are two different ways of writing the same number as you see from the following examples:

a) **Both read "5 tenths"**

$$\underset{\text{tenths}}{0.5} = \frac{5}{10}$$

b) **Both read "4 and 5 hundredths"**

$$\underset{\text{hundredths}}{4.05} = 4\frac{5}{100} \longleftarrow \text{a mixed number}$$

In decimal fractions, a power of 10 (10, 100, etc.) is shown in the denominator. In decimals, a power of 10 is not shown but is indicated by the position of the digit after the decimal point.

c) **Both read "6 thousandths"**

$$\frac{6}{1000} = 0.006$$

Since 6 is in thousandths place (3rd place) use zeros to hold the tenths & hundredths places.

d) **Both read "2 and 6 tenths"**

$$2\frac{6}{10} = 2.6 \longleftarrow \text{a mixed decimal}$$

The decimal point is for the word "and."
$2.6 = 2 + 0.6$

Changing Decimal Fractions To Decimals: A How-To (See also p.384)

Shortcut: Move the decimal point in the numerator to the left as many places as there are zeros in the denominator. The reason is that **fraction means division.**
Example: Write 642/1000 as a decimals. (See also p.229)

$$\frac{642}{1000} = 642 \div 1000 = .642$$

See "Dividing Decimals by 10, 100, etc".

Changing Decimals To Decimal Fractions: A How-To (See also p.385)

Shortcut: Write the decimal digit as the numerator over 1. Then add to 1 the number of zeros that equal to the number of decimal places. Here is the reason behind the shortcut.
Example: Write .25 as a decimal fraction. (See also p.228)

$$\overset{(1)}{\frac{.25}{1}} = \overset{(2)}{\frac{.25 \times 100}{1 \times 100}} = \overset{(3)}{\frac{25}{100}}$$

The number of zeros in the denominator equals the number of decimal places.

(1) Write the decimal as a fraction - the decimal over 1.
(2) Multiply the fraction by 100/100 to get rid off the decimal point.
(3) The decimal fraction 25/100 is an equivalent fraction of .25/1.

Comparing and Ordering Fractions (Read "Equivalent Fractions" p.300)

Fractions can be compared **only** when they have the **same denominators.** For how to change unlike fractions to like fractions see pages 320-325.

1. To compare fractions with **like denominators:** 2/5 and 4/5

$$\frac{2}{5} \qquad \frac{4}{5}$$

If the fractions have the same denominators, **the greater the numerator, the greater the fraction.** Since 4 > 2, 4/5 > 2/5.

2. To compare fractions with **like numerators:** 7/9 and 7/12

$$\frac{7}{9} \qquad \frac{7}{12}$$

If the fractions have the same numerator, **the greater the denominator, the smaller the fraction.** Since 12 > 9, 7/12 < 7/9.

3. To compare fractions with **different denominator:** 3/4 and 5/8

$$\frac{3}{4} \underset{\times 2}{\overset{\times 2}{=}} \frac{6}{8}, \qquad \frac{5}{8}$$

1st. Change unlike fractions to like fractions by using **equivalent fractions.**
2nd. Compare the numerators of the fractions.
Since 6 > 5, 3/4 > 5/8

4. To compare **mixed numbers:** 1 2/3 and 2 7/8

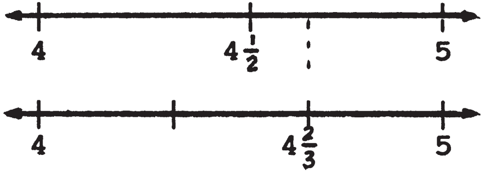
$1\frac{2}{3}, \qquad 2\frac{7}{8}$

Compare the **whole number part first.**
Since 1 < 2, 1 2/3 < 2 7/8.

5. To compare **mixed numbers:** 4 1/2 and 4 2/3

$4\frac{1}{2}, \quad 4\frac{3}{6}, \quad 4\frac{2}{3}, \quad 4\frac{4}{6},$

Since the whole number part are equal, **compare the fractional part.**



Or compare them on **number lines.**
The number on the right is always greater than the number on the left.
So, 4 2/3 > 4 1/2.

6. **To order the numbers** from least to greatest: 3 2/3, 3 2/9, 3 7/9

$$\begin{tikzcd}\frac{2}{3} \arrow[r, bend left, "\times 2"] \arrow[r, bend right, "\times 2"'] \arrow[r, phantom, "="] & \frac{6}{9},\end{tikzcd} \qquad \frac{2}{9}, \qquad \frac{7}{9}$$

Compare the whole numbers part first.
Then compare the fractional part.
Order from least to the greatest:
3 2/9 < 3 2/3 < 3 7/9

Summary
(Fractions)

* Fractions are used to express "a part of the whole," or "ratios," or "division." By its context, you can tell which one of the meanings is more relevant to the situation.
* A fraction is made up of two numbers - the numerator and the denominator - which are divided by a horizontal line called a fraction bar
* All fractions signify division. The numerator (top number) is the dividend and the denominator (bottom number) is the divisor
* Fractions can be classified as simple and complex, proper and improper, like and unlike fractions.
* Improper fractions and mixed numbers are interchangeable. Mixed numbers are one of the three ways of writing quotients with remainders in whole number division.
* Decimal fractions can easily be changed to decimals and vice versa.

Part II. Whole Number Operations

A. Introduction

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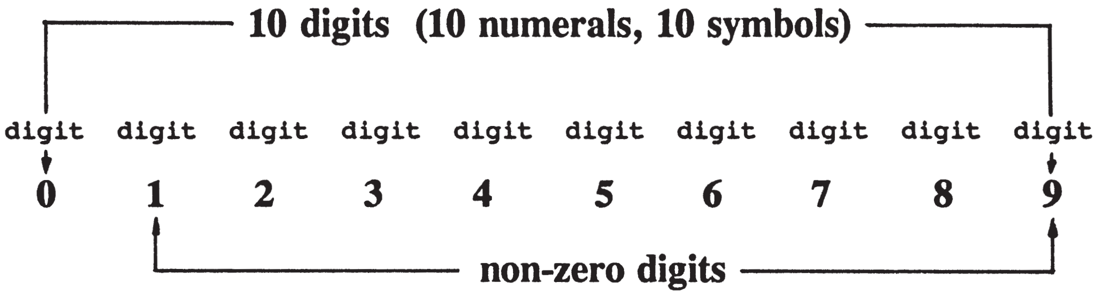
Digit, Ten Digits



Examples:

10 is a two-digit number.
596 is a **three**-digit number.
27,081 is a **five**-digit number.

Remember, 10 is not a digit.

Connection: We use the ten digits to write numbers, just as we use the twenty-six letters of alphabet to write words. In fact, we can use these ten digits and **a decimal point** to write very large numbers and very small numbers. Remember, we can't write the largest or the smallest numbers. Do you know why? Think!

Meanings Of The Symbol "0"

In the beginning, we were taught that the big "0" (zero) means "nothing." Well, it really depends on where and how the symbol 0 is used. At least, it has the following meanings:

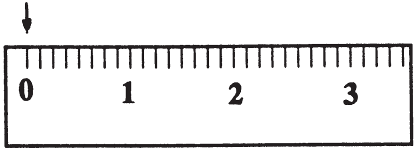
1. $18 - 18 = \overset{\downarrow}{0}$ $\quad 7 + \overset{\downarrow}{0} = 7$

Zero means **"no thing" (nothing)** when it stands by itself.

2. $5\,\overset{\downarrow}{0}\,9$ $\quad 8\,\overset{\downarrow}{0}\,\overset{\downarrow}{0}\,1$

$\boxed{\textbf{place holders}}$

Zero serves as a **"place holder"** in a number. **When "0s" are used as a place holders, they can not be omitted.**

3. 

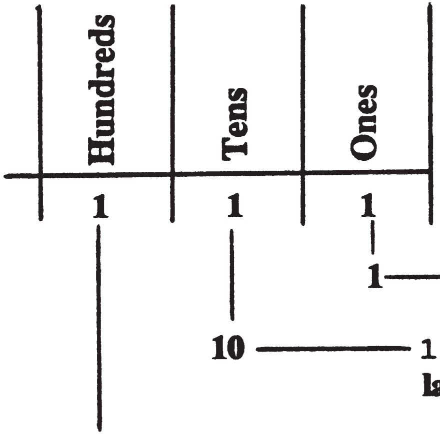
Zero marks the **"beginning"** in all measurement -- such as rulers, measuring tapes, scales, etc.

4. 

Zero is called **"origin"** on a number line and a coordinate graph.

Decimal System Or Base-10 System (Read also p.16)

Our number system is called decimal system. The word "deci" means "ten." According to the decimal system, **"The value of any place (or digit) is 10 times larger than the place (or digit) to its right."** A place value chart can help us understand how the decimal system works:



You can find the "Place Value Chart of Whole Numbers" on page 18.

1——1 in the "***ones*** **place**" equals **1**.

10 ———— 1 in the "***tens*** **place**" equals **"10 ones"** - **10 times larger** than the 1 in the "ones place" (1 x 10 = 10).

100 ———— 1 in the "***hundreds*** **place**" equals **"10 tens"** **10 times larger** than the 1 in the "tens place" (10 x 10 = 100).

Do you know **carrying** and **borrowing** has to do with decimal system? (p.140)

Place Value System (Read also p.17)

The decimal system implies the place value system. Place value system says, **"Every digit in a number has a value, and the value of a digit depends on its place or position in the number."** For example, the number 3 has a different value in each of the following numbers: 3, 39, and 326.

Hundreds	**Tens**	**Ones**	
		③	**3** ***ones*** (Read "three")
	③	9	**3** ***tens*** 9 ones (Read "thirty-nine")
③	2	6	**3** ***hundreds*** 2 tens 6 ones (Read "three hundred twenty-six")

Do you see that the number 3 has a different value in each of the numbers above? The value of the number 3 depends on its place in the number. Therefore, **be conscious of place value when you work with numbers!**

Four Basic Operations **(See next page for the definition of the terms.)**

The four basic operations given below may be considered the foundations of arithmetic. We always **begin with addition,** then subtraction, multiplication, and finally division. The reason for that order is:

- Subtraction is the *inverse operation* of addition. **(See p.150)**
- Multiplication is a *repeated addition*. **(See p.168)**
- Division is a *repeated subtraction*. **(See p.192)**, and the *inverse operation* of multiplication. **(See p.193)**

Addition

6	**addend**
+ 8	**addend**
14	**sum**

Subtraction

14	**minuend**
- 8	**subtrahend**
6	**difference**

Multiplication

5	**multiplicand (factor)**
x 9	**multiplier (factor)**
45	**product (multiple)**

Division

9	**quotient (factor)**
5) 45	**dividend (multiple)**
	divisor (factor)

Mathematics, like every other subject, ***has its own vocabulary.*** If you want to understand the language of the mathematics book, you have to learn ***mathematics terms and expressions.*** The following terms are related to four basic operations. They are used throughout the book. **Memorize them!**

* **Addends** - the numbers that you add together to get sum.
Sum - the answer of adding two or more numbers together.

* **Minuend** - the number from which you subtract (top number).
Subtrahend - the number you use to subtract (bottom number).
Difference (or **remainder**) - the result of subtraction. (The answer)

In subtraction with whole numbers, the minuend is always larger than the subtrahend. (See p.122)

* **Multiplicand** - the number to be multiplied.
Multiplier - the number you use to multiply the multiplicand.
Product (or **multiple**) - the result of multiplication. (The answer)
Factors - the numbers you multiply together to get product.

Factors are another name for the multiplicand, multiplier, divisor, and quotient. (See p.198)

* **Dividend** - the number to be divided into equal part.
Divisor - the number used to divide the dividend.
Quotient - the result of division. (The answer)

In division with whole numbers, the dividend is always larger than the divisor. (See p.122)

Operation Symbols & Key Words

To translate word problems into number sentences (or mathematic equations) is an important part of mathematics study. If you are familiar with the following key words/phrases, it will help you to choose the **correct operation** for solving problems.

Addition: (+)
* 8 **plus** 2 (8 + 2)
* **Add** 6 and 3 (6 + 3)
* 4 **more than** 3 (3 + 4)
* **The sum of** 5 and 7 (5 + 7)
* 2 **increased by** 6 (2 + 6)
* **how many (much) altogether**
* **the total of, in all**

Subtraction: (–)
* 7 **minus** 3 (7 - 3)
* **Subtract** 4 from 9 (9 - 4)
* 10 **less** 5 (10 - 5)
* **The difference of** 8 and 2 (8 - 2)
* 6 **decreased by** 4 (6 - 4)
* **Take away** 3 from 5 (5 - 3)
* **How many (much) are left**

Multiplication: (x)
* 4 **times 6** (4 x 6)
* 5 **multiplied by** 3 (5 x 3)
* **The product of** 2 and 9 (2 x 9)

Division: (÷, $\overline{)\quad}$, /)
* 12 **divided by** 3 (12 ÷ 3)
* **The quotient of** 8 divided by 2
* **Each**

Remember the phrases: **"the sum"**, **"the difference"**, **"the product"**, and **"the quotient"**.

Multiplication Symbols & Division Symbols

It is important for you to know that we can use one of the following symbols to express multiplication or division:

Multiplication Symbols

a. **"x"** (cross-sign)
Examples. 3 x 6, 2.5 x 0.7, etc.
b. **( )** (parentheses)
Examples: 4(2.5), (1/3)(5/6), etc.
c. **"·"** (raised dots)
Examples. 3·6, a·a·a, etc.
d. **"of"** (the word "of")
Example: a quarter of a day,

Division Symbols

a. "$\div$"
Example: $9 \div 3$, $1\ 8 \div 3$
b. "$\overline{)\ \ }$"
Example: $3\,\overline{)9}$, $3\,\overline{)1\,8}$
c. " / " (fraction bar)
Example: $9/3 = 9 \div 3$
All fractions mean division.

Note: To multiply, we write the problem in **vertical form**
Note: "$\overline{)\ \ }$" is the form we use in dividing **whole numbers** and **decimals.**
Since division is not commutative, it is important that you set up the division problem correctly:
- In $9 \div 3$, the number follow "$\div$" is the divisor: $3\,\overline{)9}$
- In 2/5, the denominator is the divisor: $5\,\overline{)2}$

Checking Addition OR Subtraction

To Check Addition: **Reverse the order of addends and add.**

Add

$$\begin{array}{r} 412 \\ +\ 375 \\ \hline 787 \end{array}$$

Check

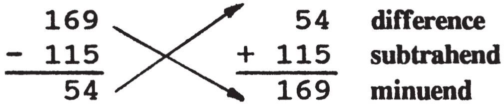
$$\begin{array}{r} 375 \\ +\ 412 \\ \hline 787 \end{array}$$

If added down, you check by adding up because addition is **commutative:**

412 + 375 = 787 (added down)
375 + 412 = 787 (added up)

To Check Subtraction: The formula is **(Difference) + (Subtrahend) = (Minuend)**

Subtract **Check**



169 - 115 = 54

To check subtraction, add the difference to the subtrahend. The sum **should equal** the minuend.

We use addition to check subtraction because they are **inverse operations.**

Checking Multiplication OR Division

To Check Multiplication: Reverse the order of the factors and multiply.

Multiply

$$\begin{array}{r} 24 \\ \text{x}\ 15 \\ \hline 360 \end{array}$$

Check

$$\begin{array}{r} 15 \\ \text{x}\ 24 \\ \hline 360 \end{array}$$

Since multiplication is **commutative,** you can multiply in **different order:**

$24 \text{ x } 15 = 360$

$15 \text{ x } 24 = 360$

To Check Division: The formula is (Quotient) x (Divisor) = (Dividend)

Divide

$$\begin{array}{r} 24 \\ 15\ \overline{)\,360} \\ -\ 360 \\ \hline \end{array}$$

$360 \div 15 = 24$

Check

$$\begin{array}{rl} 24 & \text{quotient} \\ \text{x}\ 15 & \text{divisor} \\ \hline 360 & \text{dividend} \end{array}$$

To check the division, multiply the quotient by the divisor, the product **should equal** the dividend.

We use multiplication to check division because they are **inverse operations.** **(See also p.198)**

Estimating Sums of Whole Numbers (Review first "Rounding Off" p.30)

There are different ways of estimating a sum. The following shows two commonly used methods. Estimate 875 + 1,219 + 2,042:

Write the numbers vertically according to their place values will help you to determine the front or lead digit of a number Not every first digit of a number is a lead digit like 875 in the following example.

Method 1. Using Front-end Digits

Step 1. Get a **rough estimate** by **adding only the front digits.** "0s" for the other digits.

Step 2. Adjust the estimate (Step 1) by adding an estimated sum of the remaining digits: 875, 219, 42 is about 1,000.

$$\begin{array}{r} 875 \\ 1,219 \\ +\ 2,042 \\ \hline 3,000 \end{array} \qquad \text{about } 1,000$$

3,000 + 1,000 = **4,000**

Method 2. Using Rounding

Round each number to the **same place** that can be added easily. So, round to the largest place and add.

$$\begin{array}{rcr} 875 & \longrightarrow & 1,000 \\ 1,219 & \longrightarrow & 1,000 \\ +\ 2,042 & \longrightarrow & 2,000 \\ \hline 4,136 & & 4,000 \\ \text{actual sum} & & \text{estimate sum} \end{array}$$

Remember: Use front-end digits estimation (Step 1) without adjustment (step 2) will lose too much especially when the numbers are large. Adjusting the estimate is an attempt to get closer to the actual sum or difference.

Estimating Differences of Whole Numbers (Review first "Rounding Off" p.30)

The following shows how to estimate the difference of 24,249 - 4,888:

The front digit (4) of the smaller number (4,888) is the lead digit.

$$\begin{array}{r} 21{,}249 \\ \underline{-\ 4{,}888} \end{array} \qquad \begin{array}{r} 21{,}000 \\ \underline{-\ 4{,}000} \\ 17{,}000 \end{array}$$

Method 1. Using Front-end Estimation

Step 1. Get a **rough estimate** by subtracting the front digits: 17,000

Since 888 > 249,
the difference is < 17,000

Step 2. Compare the remaining digits and **adjust** the difference.

$$\begin{array}{rcr} 21{,}249 & \longrightarrow & 20{,}000 \\ \underline{-\ 4{,}888} & \longrightarrow & \underline{-\ 0{,}000} \\ & & 20{,}000 \end{array}$$

Method 2. Using Rounding

Round each number to the **same place**, the largest place: ten-thousands.

The above estimate is **not reasonable** because 4,888, the subtrahend, was rounded down to 0. Try again.

$$\begin{array}{rcr} 21{,}249 & \longrightarrow & 21{,}000 \\ \underline{-\ 4{,}888} & \longrightarrow & \underline{-\ 5{,}000} \\ & & 16{,}000 \end{array}$$

So, round to the next largest place: to the nearest thousands.

← This estimate is **more reasonable.**

Estimating Products of Whole Numbers (Review first "Rounding Off" p.30)

There are many ways to estimate a product. Choose a method that will give a closer estimate for the situation. Each method involves multiplying the multiples of 10, 100, etc. (See also p.180)

$$\begin{array}{r} 34 \\ \underline{\times\ 27} \\ \end{array} \qquad\qquad \begin{array}{r} 30 \\ \underline{\times\ 20} \\ 600 \end{array}$$

Method 1. Using Front Digits
Multiply only the front digits. The estimate is always **an underestimate.**

$$\begin{array}{rcccr} 34 & \longrightarrow & \text{down} & \longrightarrow & 30 \\ \underline{\times\ 27} & \longrightarrow & \text{up} & \longrightarrow & \underline{30} \\ & & & & 900 \end{array}$$

Method 2. Rounding One down & One Up
Round one factor down, the other up and multiply. It gives a **closer estimate.**

$$\begin{array}{rcr} \multicolumn{3}{c}{\textbf{Round down}} \\ 34 & \rightarrow & 30 \\ \underline{\times\ 27} & \rightarrow & \underline{20} \\ & & 600 \end{array} \qquad\qquad \begin{array}{rcr} \multicolumn{3}{c}{\textbf{Round up}} \\ 34 & \rightarrow & 40 \\ \underline{\times\ 27} & \rightarrow & \underline{30} \\ & & 1200 \end{array}$$

Method 3. Find the Range of a Product
Rounding **both factors** ***up***, and **both** ***down*** gives **the range of the product.** The actual product will be **between** 600 **and** 1200.

$$\begin{array}{rcr} 572 & \longrightarrow & 600 \\ \underline{\times\ \ 63} & \longrightarrow & \underline{60} \\ & & 3600 \end{array}$$

Method 4. Rounding to the Largest Place
Round each number to **its largest place, not the same place**, and multiply.

Estimating Quotients of Whole Numbers

One way to estimate quotients is to use the multiplication/division facts and powers of 10. We change the given dividend, or divisor, or both to a pair(s) of multiplication fact(s) which are **compatible to the given numbers** so that we can **divide mentally.**

a) 227 ÷ 4 ┤ 200 ÷ 4 = 50 / 240 ÷ 4 = 60

Since 5 x 4 = 20 & 6 x 4 = 24, **change** 227 to 200 (20 ÷ 4) or 240 (24 ÷ 4) - both numbers are close to 227. The actual quotient is between 50 and 60.

b) 472 ÷ 55 ┤ 450 ÷ 50 = 9 / 500 ÷ 50 = 10

Since 9 x 5 = 45 & 10 x 5 = 50, **change** 55 to 50 and change 472 to 450 or 500 -- both numbers are close to 472.

472 ÷ 55 480 ÷ 60 = 8

Or **change** 55 to 60 and change 472 to 480 because 48 ÷ 6 = 8.

Note: The above examples show that there can be **more than one** reasonable estimates because there are more than one pair of compatible numbers. However, you should have some idea whether **the estimate is an underestimate or overestimate and know the reason why.**

Four Basic Operations & Properties

The properties of mathematics are something that are always true and never changed. If you want to do well in math, it is **a must** that you study carefully the properties and learn how to use them to simplify computations. Some of the basic properties are listed below for your convenience. To know how a property is used, please turn to the indicated page. In order to write these properties in general form, we use **a**, **b**, and **c**, to represent **any whole numbers.**

Addition Properties:

1. **Identity (Zero) Property:** $a + 0 = a$
   The sum of a number and zero is that number. **(See p.130)**
2. **Commutative (Order) Property:** $a + b = b + a$
   Changing the order of the addends does not change the sum. **(See p.130)**
3. **Associative (Grouping) Property:** $(a + b) + c = a + (b + c)$
   Re-grouping of addends does not affect the sum. **(See p.134)**

Subtraction Property of Zero:

1. The difference of a number and zero is the number: $a - 0 = a$ **(See p.153)**

2 The difference of a number and itself is zero: $a - a = 0$ **(See p.153)**

Connection: These properties are also applied to ***Decimals*** & ***Fractions.***

Multiplication Properties:

1. **Zero Property: $a \times 0 = 0$**
   The product of any number and zero is zero. **(See p.172)**
2. **Identity (One) Property: $a \times 1 = a$**
   The product of any number and one is that number. **(See p.172)**
3. **Commutative (Order) Property: $a \times b = b \times a$**
   Changing the order of factors does not affect the product. **(p.172)**
4. **Associative (Grouping) Property: $(a \times b) \times c = a \times (b \times c)$**
   Re-grouping of factors does not affect the product. **(p.202)**
5. **Distributive Property of Multiplication:** **(See p.137)**
   $a \times (b + c) = (a \times b) + (a \times c)$ **and** $(b + c) \times a = (b \times a) + (c \times a)$
   $a \times (b - c) = (a \times b) - (a \times c)$ **and** $(b - c) \times a = (b \times a) - (c \times a)$

Division Properties:

1. **Zero Property: $0 \div a = 0$** (a can not be zero) **(See p.195)**
   The quotient of zero divided by any number is zero. **(See p.196)**
2. **One Property: $a \div a = 1$** (a can not be zero)
   The quotient of any number divided by itself is one. **(See p.196)**
3. **Identity Property: $a \div 1 = a$**
   The quotient of any number divided by one is that number. **(p.196)**
4. **Division by zero: $a \div 0$ & $0 \div 0$ is impossible!** **(See p.195)**

Connection: One property of multiplication/division is important in fractions.

Special Rules For Subtraction And Division

Keep the following rules in mind when you do **subtraction** and **division.**

1. In **subtracting** *whole numbers, decimals, and fractions,* we always **subtract a smaller number from a larger number.** For example:

   $7 - 4 = 3$ and $4 - 7 =$ **impossible** (because $4 - 7 = -3$)

   $3.5 - 2.1 = 1.4$ and $2.1 - 3.5 =$ **impossible** ($2.1 - 3.5 = -1.4$)

   $5/7 - 2/7 = 3/7$ and $2/7 - 5/7 =$ **impossible** ($2/7 - 5/7 = -3/7$)

   If we subtract a larger number from a smaller number, the answer will be a "**negative**" number. In arithmetic, we are dealing only with "**positive**" numbers.

2. In **dividing** *whole numbers,* the divisor must be smaller, or at most equals to, the dividend. For example:

   $5 \div 4 = 1$ r1 and $4 \div 5 =$ **impossible** (because $4 \div 5 = 0.8$)

   If we divide a smaller number by a larger number like $4 \div 5$, the answer will not be a "**whole number**" but a "**decimal**", less than 1.

Binary Operation & Basic Facts

A binary operation is an operation that performs on two numbers at a time. **The four basic operations are binary operations, which means you add, subtract, multiply, or divide only two numbers at a time.** See the examples below:

1. Add. 8 + 6 + 7 + 9 + 5
   First. You add two numbers, any two, together: **8 + 6** = 14
   Then add another number to the sum of 14: **14 + 7** = 21
   And continue to add another number to the new sum,...

2. Subtract. $\begin{array}{r} 69 \\ -\ 25 \\ \hline 44 \end{array}$
   1st. You subtract the ones digits: **9 - 5** = 4.
   2nd. You subtract the tens digits: **6 - 2** = 4.
   (6 - 2 = 4 is actually 60 - 20 = 40)

2. Multiply. $\begin{array}{r} 521 \\ \times\ \ 4 \\ \hline 2084 \end{array}$
   1st. You multiply **1 x 4** = 4.
   2nd. You multiply **2 x 4** = 80 (actually 4 x 20)
   3rd. You multiply **5 x 4** = 2000 (actually 4 x 500)

Look over the examples again, do you see that we worked on two numbers at a time, and each time we used the basic facts? The fact is, **you will use the basic facts to compute all** addition, subtraction, multiplication, and division problems, no matter how large the numbers may be.

Summary

(Introduction)

* We use the ten digits, with zeros (0s) as place holders, to write numbers. When zeros are used as place holders, they can not be omitted.
* With a sound understanding of the decimal system/place value system, many arithmetic works become easy.
* The four basic operations - addition, subtraction, multiplication, and division - are different ways of counting and are related. For example, We can use addition to check subtraction, and use multiplication to check division.
* Knowledge of mathematical vocabulary and symbols are essential in studying math.
* Properties are the rules of mathematics. They are used to simplify computations. Learn those side by side with the four basic operations and use them to your advantage.
* Rounding is a common skill used to estimate the answers of computations.

Part II. Whole Number Operations

B. Addition

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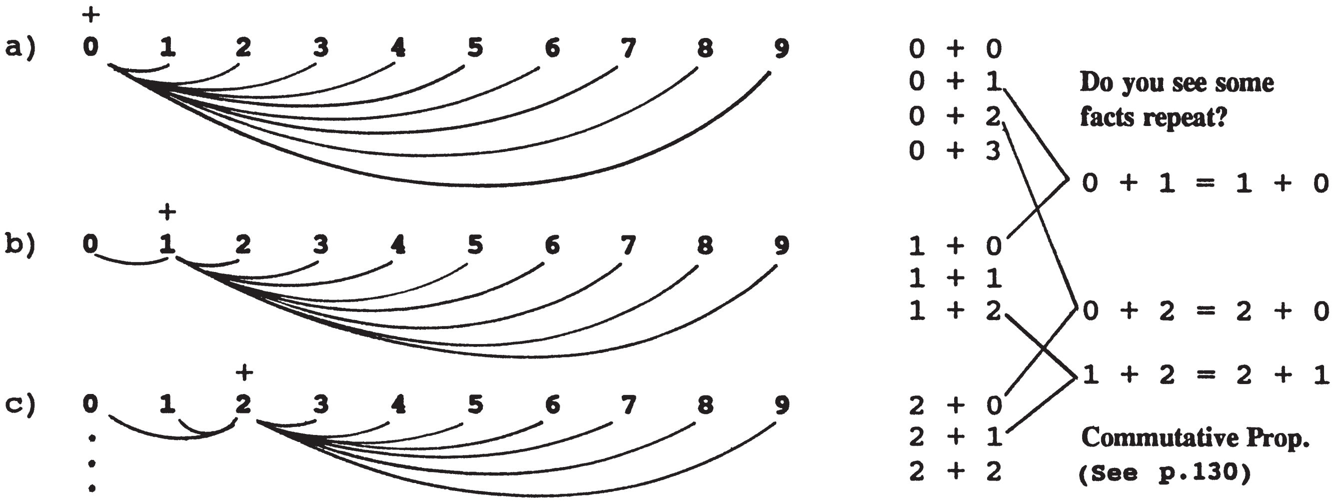
Addition Facts: Combinations of Ten Digits (See "Ten Digits" p.106)

The following shows how we got our addition facts. They are the combinations of the ten digits (0 to 9). Start with 0. Add 0 to itself, then added to each of the remaining digits, one at a time in order, as seen in a). Follow the same procedure for each digit, 1 through 9, and you will have the 100 basic addition facts found on the next page.



Do you see some facts repeat?

Commutative Prop. (See p.130)

Note: The sign "+" serves to remind you that you always add the number to itself.

100 Basic Addition Facts - One-Digit Addition

0 + 0 = 0
0 + 1 = 1
0 + 2 = 2
0 + 3 = 3
0 + 4 = 4
0 + 5 = 5
0 + 6 = 6
0 + 7 = 7
0 + 8 = 8
0 + 9 = 9

1 + 0 = 1
1 + 1 = 2
1 + 2 = 3
1 + 3 = 4
1 + 4 = 5
1 + 5 = 6
1 + 6 = 7
1 + 7 = 8
1 + 8 = 9
1 + 9 = 10

2 + 0 = 2
2 + 1 = 3
2 + 2 = 4
2 + 3 = 5
2 + 4 = 6
2 + 5 = 7
2 + 6 = 8
2 + 7 = 9
2 + 8 = 10
2 + 9 = 11

3 + 0 = 3
3 + 1 = 4
3 + 2 = 5
3 + 3 = 6
3 + 4 = 7
3 + 5 = 8
3 + 6 = 9
3 + 7 = 10
3 + 8 = 11
3 + 9 = 12

4 + 0 = 4
4 + 1 = 5
4 + 2 = 6
4 + 3 = 7
4 + 4 = 8
4 + 5 = 9
4 + 6 = 10
4 + 7 = 11
4 + 8 = 12
4 + 9 = 13

5 + 0 = 5
5 + 1 = 6
5 + 2 = 7
5 + 3 = 8
5 + 4 = 9
5 + 5 = 10
5 + 6 = 11
5 + 7 = 12
5 + 8 = 13
5 + 9 = 14

6 + 0 = 6
6 + 1 = 7
6 + 2 = 8
6 + 3 = 9
6 + 4 = 10
6 + 5 = 11
6 + 6 = 12
6 + 7 = 13
6 + 8 = 14
6 + 9 = 15

7 + 0 = 7
7 + 1 = 8
7 + 2 = 9
7 + 3 = 10
7 + 4 = 11
7 + 5 = 12
7 + 6 = 13
7 + 7 = 14
7 + 8 = 15
7 + 9 = 16

8 + 0 = 8
8 + 1 = 9
8 + 2 = 10
8 + 3 = 11
8 + 4 = 12
8 + 5 = 13
8 + 6 = 14
8 + 7 = 15
8 + 8 = 16
8 + 9 = 17

9 + 0 = 9
9 + 1 = 10
9 + 2 = 11
9 + 3 = 12
9 + 4 = 13
9 + 5 = 14
9 + 6 = 15
9 + 7 = 16
9 + 8 = 17
9 + 9 = 18

Addition Properties & Addition Facts (Review first "Properties" p.120)

It is **very** important that you memorize the addition facts because all future addition problems and other operations, depend on these basic facts. The following shows that by using **two addition properties,** you can cut down the memory work by half.

$0 + 1 = 1$ or $1 + 0 = 1$
$0 + 2 = 2$ $2 + 0 = 2$
$0 + 3 = 3$ $3 + 0 = 3$
$0 + 4 = 4$ $4 + 0 = 4$
$0 + 5 = 5$ $5 + 0 = 5$

The **zero property** of addition says, **"Any number plus 0 is the number."** If you know this property, you can omit 19 facts from the list - they are the ones inside the boxes. (See p.129)

$0 + 1 = 1 + 0$
$3 + 5 = 5 + 3$
$4 + 7 = 7 + 4$
$6 + 8 = 8 + 6$
$9 + 2 = 2 + 9$

The **Commutative property** of addition says, **"The order in which numbers are added does not affect the result."** Again, by using this property, we cut down the numbers of facts to be memorized by half. (See next page.)

Remember: These two addition properties apply also to other addition problems including decimals and fractions.

Rearranging The Addition Facts

By using the **zero property** and the **commutative property** of addition, the numbers of addition facts to be learned are reduced to 45 as listed below. In practicing, **make it a habit** of saying each addition facts **both ways** before giving the sum like this: **3 + 5 = 5 + 3 = 8** or **3 + 5 equals 5 + 3 is 8.**

* 1 + 1 = 2
1 + 2 = 2 + 1 = 3
1 + 3 = 3 + 1 = 4
1 + 4 = 4 + 1 = 5
1 + 5 = 5 + 1 = 6
1 + 6 = 6 + 1 = 7
1 + 7 = 7 + 1 = 8
1 + 8 = 8 + 1 = 9
1 + 9 = 9 + 1 = 10
* 2 + 2 = 4
2 + 3 = 3 + 2 = 5
2 + 4 = 4 + 2 = 6
2 + 5 = 5 + 2 = 7
2 + 6 = 6 + 2 = 8
2 + 7 = 7 + 2 = 9
2 + 8 = 8 + 2 = 10
2 + 9 = 9 + 2 = 11
* 3 + 3 = 6
3 + 4 = 4 + 3 = 7
3 + 5 = 5 + 3 = 8
3 + 6 = 6 + 3 = 9
3 + 7 = 7 + 3 = 10
3 + 8 = 8 + 3 = 11
3 + 9 = 9 + 3 = 12
* 4 + 4 = 8
4 + 5 = 5 + 4 = 9
4 + 6 = 6 + 4 = 10
4 + 7 = 7 + 4 = 11
4 + 8 = 8 + 4 = 12
4 + 9 = 9 + 4 = 13
* 5 + 5 = 10
5 + 6 = 6 + 5 = 11
5 + 7 = 7 + 5 = 12
5 + 8 = 8 + 5 = 13
5 + 9 = 9 + 5 = 14
* 6 + 6 = 12
6 + 7 = 7 + 6 = 13
6 + 8 = 8 + 6 = 14
6 + 9 = 9 + 6 = 15
* 7 + 7 = 14
7 + 8 = 8 + 7 = 15
7 + 9 = 9 + 7 = 16
* 8 + 8 = 16
8 + 9 = 9 + 8 = 17
* 9 + 9 = 18

Note: The addition facts with * have identical addends.

Addition Table

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

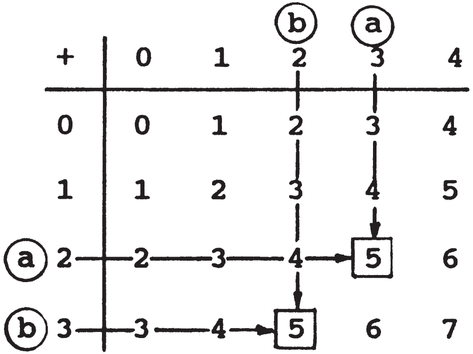
← **Addends**

↑ **Addends**

Study the table carefully. Do you see the addition properties? Do you see different pairs of numbers that add up to the same number?

Using The Addition Table To Find Sums

In the addition table, the digits at the top and the digits at the left side are addends. There are **two ways** of looking up the sum of two numbers because ***additions are commutative***. See the example below:



	+	0	1	ⓑ 2	ⓐ 3	4
	0	0	1	2	3	4
	1	1	2	3	4	5
ⓐ	2	2	3	4	5	6
ⓑ	3	3	4	5	6	7

Finding The Sum: 3 + 2 = 2 + 3 = □

- * Find 3 in the top row ⓐ , and 2 in the left column ⓐ . Then go down the column of 3 and go across the row of 2. **The sum is the number where the column and the row meet.** Or
- * Find 3 in the left column ⓑ , and 2 in the top row ⓑ . **The sum is where the column and the row meet.**

Patterns In Addition Table

Study carefully the addition table and see whether you can find the following:

* Do you see all the addition properties?
* Do you see the different pairs of numbers that give the same sum?

Numbers Adding Up To 10 (See also "Subtracting From 10" p.155)

When adding a column of numbers, **always look for numbers, two or three, that add up to 10.** It helps to speed up the computations. See the examples below:

a) $$\begin{array}{r} 4 \\ 3 \\ 6 \\ +\ 7 \\ \hline 20 \end{array}$$

b) $$\begin{array}{r} 7 \\ 5 \\ 2 \\ +\ 1 \\ \hline 15 \end{array}$$

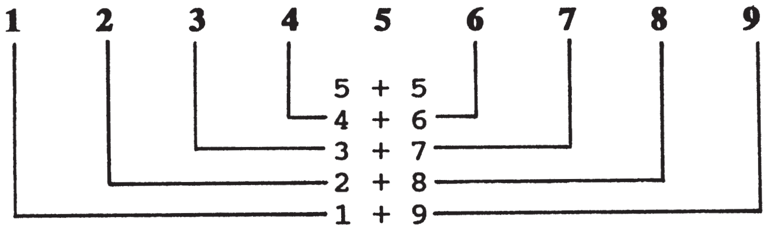
c) $$\begin{array}{r} 16 \\ 6 \\ 3 \\ +\ 4 \\ \hline 29 \end{array}$$

d) $$\begin{array}{r} 1 \\ 8 \\ 9 \\ +\ 2 \\ \hline 20 \end{array}$$

In a) $4 + 6 = 10$; $3 + 7 = 10$
b) $7 + 2 + 1 = 10$
c) $16 + 4 = 20$
d) $1 + 9 = 10$; $8 + 2 = 10$
Here we use the associative property

Use the following diagram to help you **memorize** the two numbers and three numbers that add up to ten.

Two numbers add up to 10:



Three numbers add up to 10:

1 + 1 + 8 = 10
1 + 2 + 7 = 10
1 + 3 + 6 = 10
1 + 4 + 5 = 10
2 + 2 + 6 = 10
2 + 3 + 5 = 10
2 + 4 + 4 = 10
3 + 3 + 4 = 10

Applying What You Know

Put your brain to work when studying mathematics! When doing math work, take a good look at the problem first. Always look for ways that you can apply what you already know **to simplify the computation.** For example, if you know the numbers that add up to 10, then you can compute the following problems mentally.

a) If you know: 2 + 8 = 10
Then apply: 12 + 8 = 20
32 + 18 = 50 ← So, (32 + 8) + 10 = 50, or 30 + (2 + 18) = 50

Remember, you can break numbers apart!
Think: 32 = 30 + 2; 18 = 10 + 8.

b) If you know: 2 + 5 = 7
Then apply: 20 + 50 = 70
200 + 500 = 700

Check:

$$\begin{array}{r} 20 \\ +\ 50 \\ \hline 70 \end{array} \qquad \begin{array}{r} 200 \\ +\ 500 \\ \hline 700 \end{array}$$

If the addends have equal numbers of zeros at the end, add the non-zero digits and keep the number of zeros.

c) If you know: 4 + 6 = 10
Then apply: 40 + 60 = 100
400 + 600 = 1000

Check:

$$\begin{array}{r} 40 \\ +\ 60 \\ \hline 100 \end{array} \qquad \begin{array}{r} 400 \\ +\ 600 \\ \hline 1000 \end{array}$$

Playing With Numbers (I) - Breaking Numbers Apart

In the process of learning the basic facts, many students tend to see these facts as something fixed. In fact, **any digit or number larger than 1 can be broken apart into a sum of two smaller numbers.** If you know the **rules of mathematics,** you can manipulate numbers with ease. For example:

a) $2 + 5 = 7.$ **Remember 7 can also be written as: 0 + 7, 1 + 6, 3 + 4.**

$2 + 5 = 3 + \square$ **Replace 7 with 3 + 4.** Then cover 4 and ask your friend to **find the missing addend.** ***Fun!***

$2 + 5 = 5 + \square$ How about **replace 7 with 5 + 2.** Cover 2 and ask your friend to **name the property!** ***Fun!***

b) $8 + 5 = \square$ You can break either 8 or 5 apart into two numbers, so there will be **two numbers that add up to 10.** ***Speed up!***

$8 + 5 = \square$ **Think:** $8 = 3 + 5.$ So, $3 + (5 + 5) = 3 + 10 = 13$

$8 + 5 = \square$ **Think:** $5 = 2 + 3.$ So, $(8 + 2) + 3 = 10 + 3 = 13$

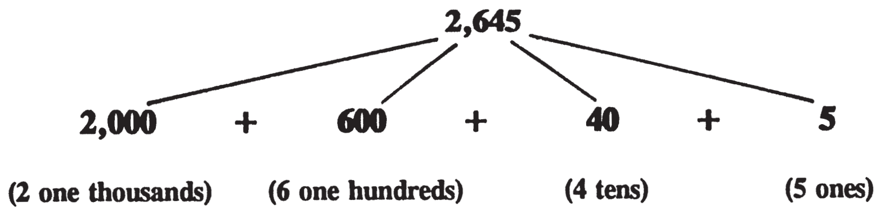
Rule: **You always compute the numbers inside the parentheses first.**
So, put the numbers that add to 10 inside the parentheses.

c) 13 − 4 = ☐	Break 13 apart. **subtract 4 from 10.** Then add the difference to the ones digit. **(See p.155)**
10 + 3 − 4 =	**Think: 13 = 10 + 3.**
(10 − 4) + 3 =	**Rearrange the numbers: Subtract 4 from 10.**
↓ 6 + 3 = 9	Add the difference to 3. ***Easy!***
d) 5 x 27 = ☐	If you know that you can break numbers apart, you can solve the problem in your head.
5 x (20 + 7) =	**Think:** 27 = **20 + 7,** because 5 x 20 is easy.
(5 x 20) + (5 x 7) =	**Distributive property over addition.**
↓ ↓ 100 + 35 = 135	Multiply 5 x 27, you will get 135.

The examples given above are only a sample of what you can do with numbers. If you know numbers can be taken apart, you can speed up computations and turn many problems into **mental math.** ***Math is fun!***

Writing Numbers in Expanded Form (See also "Expanded Form" p.24)

To write a number in expanded form is to write it **as the sum of the value of each digit.** The following shows 2645 in expanded form:



$$\begin{array}{r} 2{,}000 \\ 600 \\ 40 \\ +\quad 5 \\ \hline 2{,}645 \end{array}$$

A number can be expressed in various forms. We can write 2645

a) **in words:** **"two thousands, six hundred forty-five"**
b) **in standard numeral:** **2,645 (or 2645)**
d) **in expanded form:** **2,645 = 2,000 + 600 + 40 + 5**

It is important that you learn to read and write numbers correctly, especially large numbers. You can find the instructions on pages 18-19.

Addition Using Expanded Form (Review first the previous page.)

Writing addition problems in expanded form can help us to see the process of addition. First, write each addend in an expanded form.

Example: Add. 85 + 67

$$\begin{array}{rcr} & \xrightarrow{\quad = \quad} & \\ 8\ 5 & = & 80 + 5 \\ +\ 6\ 2 & = & \underline{60 + 2} \\ \hline 7 & & 140 + 7 = 147 \\ +1\ 4\ 0 & & \\ \hline 1\ 4\ 7 & & \end{array}$$

Writing each addend in expanded form helps us to see, that when we add **8 + 6 = 14,** we are actually adding **80 + 60 = 140** with **0 in the ones place omitted.** Do you know Why?

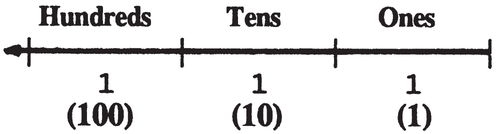
Example: Add. 8 + 97 + 103

$$\begin{array}{rcr} \prime\ \prime\ \ \ & & \\ 8 & & 8 \\ 9\ 7 & = & 90 + \ 7 \\ +1\ 0\ 3 & = & 100 + 00 + \ 3 \\ \hline 2\ 0\ 8 & & 100 + 90 + 18 = 208 \end{array}$$

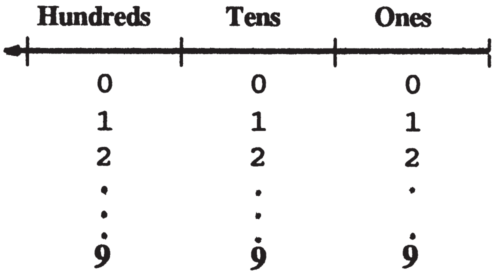
Make sure the place value of the digits are **lined up correctly.** Then,

- Add the ones: 8 + 7 + 3 = 18. ***Carry*** **one 10 to the tens place by adding 1 to the tens.**
- Add the tens: 10 + 90 = 100. ***Carry*** **one 100 to the hundreds place.**

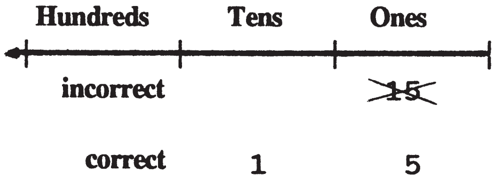
Carrying In Addition & Decimal System



Remember that in decimal system or base 10 system the value of each place is **10 times larger** than the place to its right.



According to base 10 system, each place (ones, tens, hundreds, etc.) can have ***only*** **one-digit numbers: 0, 1, 2, 3, ...** ***up to 9.*** That means we ***can not*** have **two-digit numbers** like 10, 11, 12..., etc. in any place. See the example below. Because of this, we have **"carrying"** in addition.



Therefore, in the process of adding numbers, **if the sum of any place (ones, tens, hundreds, etc.) is 10 or more,** we have to ***carry*** the 10, 20, etc. ***to the next higher place on the left.***

(Continued on the next page.)

Examples Of Carrying In Addition `(First, read the previous page.)`

(a)

$$\begin{array}{rr} & 4 \\ + & 5 \\ \hline & 9 \end{array}$$

(b)

$$\begin{array}{rrr} & & 2 \\ + & & 8 \\ \hline & 1 & 0 \end{array}$$

(c)

$$\begin{array}{rrr} & 1 & \\ & & 7 \\ + & 2 & 4 \\ \hline & 3 & 1 \end{array}$$

(d)

$$\begin{array}{rrr} & 2 & \\ & & 9 \\ & & 6 \\ + & 4 & 8 \\ \hline & 6 & 3 \end{array}$$

(e)

$$\begin{array}{rrrr} & & 5 & 5 \\ + & & 9 & 2 \\ \hline & 1 & 4 & 7 \end{array}$$

(a) Adding *ones* place: 4 + 5 = 9. ***No carrying.***

(b) Adding *ones* place: 2 + 8 = 10. ***Carrying.***
10 ones = 1 ten. So, write 1 in the tens place & 0 in the ones place.

(c) Adding *ones* place: 7 + 4 = 11. **Carrying.**
11 = 10 + 1 Carry 10 ones to the tens place by adding 1 (10 + 20 = 30)

(d) Adding *ones* place: 9 + 6 + 8 = 23. ***Carrying.***
23 = 20 + 3. Carry 20 ones to the tens place by adding 2. (20 + 40 = 60)

(e) Adding *tens* place: 5 + 9 = 14. ***Carrying.***
14 = 10 + 4. Carry 10 tens to hundreds place by adding 1

Note: We use the word "carrying" only when the place we carry to has numbers like (c) & (d).

Writing Numbers In Vertical Form (Read first "Place Value" p.18)

Math problems are often written horizontally to save space like the following examples:

(a) 2 + 4 + 6 + 8 (b) 17 + 21 + 39 + 13 (c) 15 + 7 + 243

But to compute two- or more digit numbers like (b) and (c) above, you have to rewrite the numbers vertically, one under the other. Since our number system is a place value system, **it is very important that you line up the digits correctly according to their place value.** Let's use (c) as an example.

$$\begin{array}{r} 1\ 5 \\ 7 \\ +\ 2\ 4\ 3 \\ \hline \end{array}$$

Since 15, 7, 243, each has different number of digits, digits must be lined up correctly so that you will **add the digits with the same place value.** For example:

- 5, 7, 3, are lined up in the **ones** column;
- 1, 4, are lined up in the **tens** column;
- 2 alone in the **hundreds** column.

Do you know what would happen, if you **carelessly** write 7 under 1 (tens place) instead of 5 (ones place)? 7 becomes 70, **10 times larger.** keep place value in mind when you write numbers.

General Rules For Adding Numbers

You can compute ***any*** addition problem, large and small, **with confidence, if:**

1. **You have mastered the addition facts.** **(See p.131)**
2. **You understand the place value concept.** **(See p.18)**
3. **You have learned the skill of carrying.** **(See p.140)**

The digits must be lined up correctly.

Remember to ***add*** the number that was carried over.

Adding **always begins at the ones place**, then the tens place,...

$$\begin{array}{r} {}^{1} \\ 58 \\ +\ 25 \\ \hline 83 \end{array}$$

You can **add down** (8 + 5) or **add up** (5 + 8).

Write the answer under the line. Make sure **each digit is in the right place.**

Always check your answer. If added down, check by adding up or vice versa.

Addition - Carrying More Than 10 (Read first p.140)

The **speed** and **accuracy** with which you add **depends on your knowledge of the basic addition facts.** Remember, accuracy comes first; speed second.

$$\begin{array}{r} \overset{2}{3}8 \\ 27 \\ +\ 59 \\ \hline 4 \end{array}$$

Step 1. Add ones' place.

* Add: 8 + 7 + 9 = 24 (24 = 20 + 4).

* Write 4 in the ones place under 9.
Carry 20 to the tens place by writing 2 above 3.

$$\begin{array}{r} \overset{2}{3}8 \\ 27 \\ +\ 59 \\ \hline 124 \end{array}$$

Step 2. Add tens' place.

* Add: *2* + 3 + 2 + 5 = 12 (12 means 100 + 20).
(2, above 3, is carried over from the ones place.)

* Write 2 in the tens place under 5 and 1 at the left of 2, since there is no hundreds to add to.

To Check, reverse the order (add up):

* Add ones' place: 9 + 7 + 8 = 24 ✓

* Add tens' place: 2 + 3 + 2 + 5 = 12. ✓

Remember: When adding three or more addends, you could carry more than 10 from the ones' place to the tens' place.

Addition - Carrying More Than One Place (Read first p.140)

$$\begin{array}{rccc} & & 1 & \\ & 2 & 7 & 9 \\ + & 4 & 8 & 6 \\ \hline & & & 5 \end{array}$$

Step 1. Add ones' place.

* Add: 9 + 6 = 15. (15 = 10 + 5)

* Write 5 in the ones place. *Carry 1 to the tens place.*

$$\begin{array}{rccc} & 1 & 1 & \\ & 2 & 7 & 9 \\ + & 4 & 8 & 6 \\ \hline & & 6 & 5 \end{array}$$

Step 2. Add tens' place.

* Add: *1* + 7 + 8 = 16 (16 means 100 + 60)

(1, above 7, was carried over from the ones place.)

* Write 6 in the tens place.

Carry 1 to the hundreds place.

$$\begin{array}{rccc} & 1 & & \\ & 2 & 7 & 9 \\ + & 4 & 8 & 6 \\ \hline & 7 & 6 & 5 \end{array}$$

Step 3. Add hundreds' place.

* Add: *1* + 2 + 4 = 7 (7 means 700)

(1, above 2, was carried over from the tens place.)

* Write 7 in the hundreds place under 4.

To Check, reverse the order (add up).

* Add ones' place: 6 + 9 = 15 ✓

* Add tens' place: 8 + 7 + 1 = 16 ✓

* Add hundreds place: 4 + 2 + 1 = 7 ✓

Summary
(Addition)

* Addition facts are used to compute all addition problems. These facts are also used in multiplication. Every math student should memorize them.
* From the addition table, you can find the sums of all one-digit additions, the addition properties, and different pairs of numbers that add up to the same number.
* To simplify computation, look for numbers, two or three, that add up to 10. Use what you already know to speed up your work.
* Writing numbers in expanded form can help us understand the process of adding two- or more digit numbers.
* The decimal system makes "carrying" in addition necessary At any place (ones, tens,), if the sum is 10 or more, carry the 10 to the next higher value place. Sometimes, you may have to carry 20 or more.
* To add two-or more digit numbers, write the numbers in vertical form with the value of each digit lined up correctly Then, start adding the numbers from the ones place, then the tens place, Always in that order Remember to add the numbers that are being carried over. Answer must also be lined up correctly according to their place value.
* Making it a habit of checking your answer If you added down, check by adding up. Addition is commutative.